

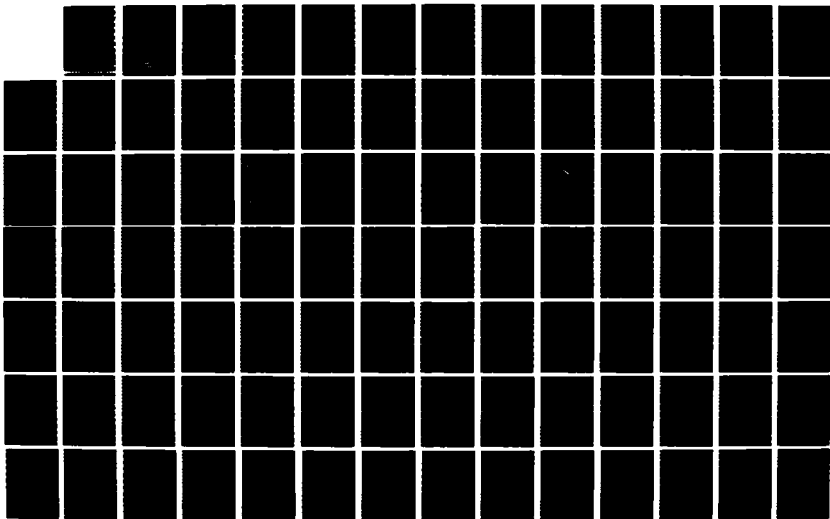
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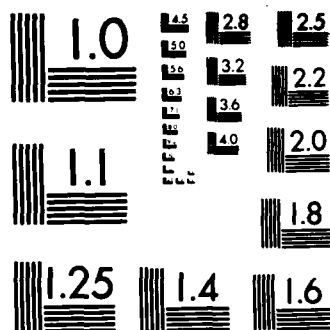
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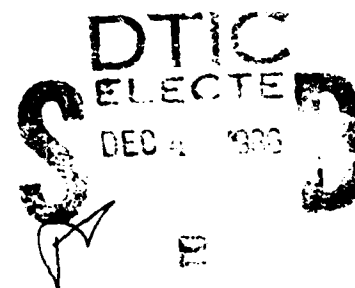
FINAL TECHNICAL REPORT  
OPTIMUM AEROELASTIC CHARACTERISTICS  
FOR  
COMPOSITE SUPERMANEUVERABLE AIRCRAFT  
(AFOSR CONTRACT F49620-85-C-0090)

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FINAL TECHNICAL REPORT  
OPTIMUM AEROELASTIC CHARACTERISTICS  
FOR  
COMPOSITE SUPERMANEUVERABLE AIRCRAFT  
(AFOSR CONTRACT F49620-85-C-0090)

July 31, 1986

Principal Investigator: Dr. G.A. Oyibo

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## 19. Abstract (cont'd)

these equations were carefully examined and the following results were revealed:

(i) incorrect modelling of the warping phenomenon can lead to errors in excess of 80% for the analytically predicted aeroelastic characteristics of composite aircraft wings, (ii) accurate modelling of the warping phenomenon is particularly important for (a) wings with mass coupling or elastic coupling (e.g., wings aeroelastically tailored using elastic coupling) and (b) higher vibration modes, (iii) neglect of the warping constraint can (a) result in either under predicted or overpredicted analytical results and (b) lead to incorrect identification of aeroelastic divergence modes. The existence of closed-form free vibrations solutions for composite wings with elastic coupling and constraint of warping was established. These closed-form solutions revealed that, (i) elastic coupling lowers the coupled frequencies (in fact a significant amount of coupling could reduce the first frequency to almost zero), (ii) it is possible to determine the origin of a coupled mode, (iii) the mechanism by which modal transformations take place may be linked to some conservative inter-modal energy transfer. It has also been found that there is an interdependency between certain design variables, which may be ordinarily taken as independent, in the tailoring technology. Influence of chord-wise bending was found to be more important for bending modes than twisting modes for orthotropic wings. A preliminary analysis seems to suggest that the constrained warping phenomenon may be captured numerically by using an "extended beam element".

# TABLE OF CONTENTS

Section		Page
1.0	ABSTRACT	1
2.0	NOMENCLATURE	3
3.0	INTRODUCTION	5
4.0	RESEARCH OBJECTIVES	11
5.0	STATUS OF RESEARCH EFFORT	13
5.1	INTRODUCTION	13
5.2	ACCURATE DYNAMIC THEORY FOR SUPERMANEUVERABLE AIRCRAFT WINGS By Gabriel A. Oyibo	15
5.3	AEROELASTIC FLUTTER PHENOMENON FOR COMPOSITE AIRCRAFT WINGS WITH WARPING CONSTRAINT By Gabriel A. Oyibo	35
5.4	CLOSED FORM DYNAMIC SOLUTIONS FOR SUPERMANEUVERABLE AIRCRAFT WINGS By Gabriel A. Oyibo	53
5.5	AEROELASTIC DIVERGENCE OF ORTHOTROPIC PLATES By Terrence A. Weisshaar	71
5.6	A STUDY OF THE EFFECTS OF PLATE STRUCTURAL IDEALIZATIONS UPON TAILORING METHODOLOGY By Terrence A. Weisshaar	85
5.7	AN EXTENDED PLATE/BEAM FINITE ELEMENT FOR COMPOSITE TAILORING STUDIES By Terrence A. Weisshaar	98
6.0	OVERALL CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK	108
7.0	STATUS OF PUBLICATIONS	110
8.0	NAME, ADDRESS AND TELEPHONE NUMBER OF PROFESSIONAL PERSONNEL	111
9.0	INTERACTIONS WITH AIR FORCE AGENCIES	112
10.0	DISCOVERIES AND INVENTIONS	113



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## 1.0 ABSTRACT

A preliminary investigation of an aeroelastically-induced constrained warping phenomenon for a composite (supermaneuverable type) aircraft wing, has been carried out in this report. The wing is analytically modelled as a straight flat laminated plate <sup>using</sup>. Various forms of highly simplified aerodynamic loads, are employed in the analysis. The free vibrations and stability aspects of this phenomenon are examined to obtain ~~some~~ physical insights and to determine ~~its~~ importance and/or design implications. Analytical tools employed include an affine transformation concept, which was formulated previously (by the Principal Investigator) as well as <sup>and</sup> a non-dimensionalization scheme. With the help of ~~these tools~~ <sup>used and</sup>, an evolution of effective warping parameters with which to study this phenomenon was <sup>performed</sup> carried out. The virtual work theorem and variational principles were used to derive the equations of motion based on the assumed wing displacements. Closed form solutions, which are generally expected to be extractable) to the uncoupled versions of these equations were carefully examined <sup>with</sup> and the following results <sup>1)</sup> were revealed: (i) incorrect modelling of the warping phenomenon can lead to errors in excess of 80% for the analytically predicted aeroelastic characteristics of composite aircraft wings; <sup>2)</sup> (ii) accurate modelling of the warping phenomenon is particularly important for (a) wings with mass coupling or elastic coupling (e.g., wings aeroelastically tailored using elastic coupling) and <sup>for</sup> (b) higher vibration modes; <sup>3)</sup> (iii) neglect of the warping constraint can (a) result in either under-predicted or overpredicted analytical results and (b) lead to incorrect identification of aeroelastic divergence modes. The existence of closed-form free vibrations solutions for composite wings with elastic coupling and constraint of warping was established. These closed-form solutions revealed that, (i) elastic coupling lowers the coupled frequencies (in fact a significant amount of coupling could

reduce the first frequency to almost zero), (ii) it is possible to determine the origin of a coupled mode, (iii) the mechanism by which modal transformations take place may be linked to some conservative inter-modal energy transfer. It has also been found that there is an interdependency between certain design variables, which may be ordinarily taken as independent, in the tailoring technology. Influence of chord-wise bending was found to be more important for bending modes than twisting modes for orthotropic wings. A preliminary analysis seems to suggest that the constrained warping phenomenon may be captured numerically by using an "extended beam element".

## 2.0 NOMENCLATURE

$a_i$	= chordwise integrals
$c'_0, c_0$	= affine space half-chord and chord, respectively
$D_{ij}$	= elastic constants
$e$	= parameter that measures the location of the reference axis relative to mid-chord
$EI, GJ$	= bending and torsional stiffness, respectively
$\bar{h}$	= wing box depth
$(h_0, \alpha_0)$	= affine space bending and torsional displacement, respectively
$K, S_0$	= elastic coupling and warping stiffness, respectively
$k_{ij}$	= elemental stiffness parameter
$k_0$	= Strouhal number
$L_0, M_0$	= affine space running aerodynamic lift and moments, respectively
$\ell_0$	= affine space half-span for the wing
$m_0$	= affine space mass per unit span
$(\Delta p, \Delta p_0)$	= differential aerodynamic pressure distributions in physical and affine space, respectively
$t$	= time
$U, U_0$	= virtual work expressions in physical and affine space, respectively
$U_f$	= flutter speed
$(x, y, z, ), (x_0, y_0, z_0)$	= physical and affine space coordinates, respectively
$\gamma_i, \delta_i, \beta_i$	= displacement shape functions

$r, L_1, L_2, D^*, D^*_0,$	= generic nondimensionalized stiffness parameters
$\mu_0$	= affine mass ratio parameter
$\mu_{ij}, \epsilon$	= Poisson ratios and generalized Poisson's ratio, respectively
$\lambda_0$	= divergence parameter
$\rho, \rho_a$	= affine space material and air density, respectively
$\theta$	= twisting displacements
$w$	= displacement
$\omega$	= vibration frequency

### 3.0 INTRODUCTION

In the late 1960's designers began to investigate the possibility of exploiting the directional properties of composite materials to improve aeroelastic characteristics of lifting surfaces. For example, superfighter designers may aeroelastically tailor a wing so that it deforms to the optimum camber under maneuvering loads.

For instance, a wing can be tailored so that its leading edge will twist downward under the stress of a tight turn, thereby decreasing the wing's angle of attack and hence reduce drag.

Exploitation of the directional properties of composite materials to solve aeroelastic instability problems (known as aeroelastic tailoring) may enhance the performance of future high performance military aircraft, particularly the super-maneuverable concepts. This stems from the need for future military weapon systems to exhibit high performance and minimum vulnerability (e.g.; minimum radar cross-sectional area). These requirements may lead to unorthodox aircraft configurations which in turn result in unorthodox aeroelastic instability problems (e.g. the X-29 primary mode of instability, called the 'body freedom' flutter).

Perhaps the aeroelastic tailoring concept would have been discovered much earlier if the physics of anisotropic aeroelasticity were more apparent. This inherent physical intractability is largely due to the existence of numerous variables, e.g.; flight parameters, several composite directional properties, fiber orientation angles, etc., which are not even truly independent, in the anisotropic system. This state of affairs is therefore analogous to that which existed in the basic rigid body aerodynamics before the advent of the similarity rule theory. This theory clearly revealed that, the utilization of

non-dimensionalized groups of variables, e.g.; Reynolds, Mach, Strouhal, Froude numbers etc., provide significantly superior physical insight to the problem than the individual physical variables. The new methodology that is being used as the basic tool in this research program is basically the aeroelastic equivalent of the aerodynamic similarity rule. The expected superiority of this new approach over the state-of-the-art (SOA) counterpart, which utilizes individual physical variables, especially in terms of physical insights, has been demonstrated in references 1-10. For example, a high-aspect-ratio composite wing could behave aeroelastically like a low aspect ratio wing and vice-versa. Similarity parameters can expose conditions for which this might happen. This is significant, (for instance) in the light of the important role played by the wing aspect ratio in the aerodynamics approximations for an aeroelastic analysis. This result may therefore be suggesting some new form of coupling between the elastic and aerodynamic equations in composite wing aeroelasticity. A fundamental aspect of this new methodology has been used in studies at Purdue University<sup>11</sup> and MIT<sup>12</sup>.

In this research, an investigation of a wing's spanwise sectional distortion (warping) resulting from aerodynamic forces and its influence on the wing's free vibrations, as well as (aeroelastic) stability, has been carried out. In particular, St. Venant's torsional/twisting theory, which is currently widely used for estimating the wing's twisting displacements has been examined with the help of the new methodology, to determine its limitations, when applied to wings fabricated of composite materials. The relevance and significance of this study for the newly emerging supermaneuverable type aircraft may be seen in the light of the fact that (a) supermaneuverability is characterized by high angle of attack which implies high twisting aerodynamic forces, and (b) most

aircraft designers believe future aircraft will be fabricated of 40-70% composite materials.

When the St. Venant's torsional theory is used to estimate a wing's twisting displacement and/or forces, the fact that the wing's root section's distortion is relatively small compared to that of other sections is ignored. However, previous investigators have determined that such an unrealistic assumption may lead to only little errors if the aspect ratio of the wing is very high.

The research has shown in this report that the conclusions reached by previous investigators are basically true for wings fabricated of metals or isotropic materials. A set of new theories is therefore postulated as shown in this report for accurately estimating the twisting displacements, vibrational frequencies and instability boundaries for wings fabricated of composite materials.

This report is divided into 8 sections: sections 1, 2 and 3 show the abstract, nomenclature, and a general introduction, respectively, while sections 4 and 5 treat the research objectives and the status of the research effort, respectively. Section 5 outlines the main findings of the research as contributed by Dr. G.A. Oyibo, of Fairchild Republic who was the Principal Investigator and Professor T.A. Weisshaar of Purdue University who participated in the research in order to broaden the research scope. The evolution of an effective warping parameter with which to study the constrained warping phenomena for wings fabricated of composite materials as well as a preliminary analysis were formulated by Dr. G.A. Oyibo and presented in sections 5.2 and 5.3. These sections basically identify the important parameters that physically define or affect these phenomena. A major accomplishment of this research program, also developed by Dr. G.A. Oyibo, was the formulation of the closed-form solutions,

for the first time, to the elastically coupled free vibration problems of composite aircraft wing undergoing constrained warping and presented in section 5.4. Professor T.A. Weisshaar has presented the effect of constrained warping on the divergence of orthotropic type wings in section 5.5, while in section 5.6 he has presented the results of his study on the interdependence between the parameters involved in aeroelastic tailoring. Professor T.A. Weisshaar has also presented the results of his preliminary investigation of the possibility of developing a numerical extended beam superelement that is capable of capturing the essence of constrained warping in aircraft wings fabricated of composite materials. The accomplishment of this task shall enable one to analyze wings of more arbitrary geometric configuration.

Section 6 provides the brief overall conclusions and recommendations for future work for furthering the understanding of these phenomena.

In Sections 7, 8, 9 and 10 are shown status of publication, the list of personnel, interactions with the Air Force agencies and discoveries and inventions, respectively.

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#### 4.0 RESEARCH OBJECTIVES

The overall goal of this research program is twofold: first, is to evolve an effective warping parameter from an advanced lamination theory, with which to study the underlying physics of the mechanism of flutter and divergence in composite aircraft wings and second, to use cantilevered flat plate models of aircraft wings along with two- and three-dimensional unsteady aerodynamic theories to conduct a study of aeroelastic stability boundaries and their dependence on the warping parameter.

The results of the efforts towards achieving this overall objective are the subject of this final report. The initial three month-effort was aimed at conducting investigations to examine various ways the warping parameter may be evolved and to determine which of these evolutions could best be utilized to effectively study the effects of the "restraint of warping" on the mechanism of vibration, aeroelastic divergence and aeroelastic flutter of aircraft wings fabricated of composite materials. The two main tools employed in conducting these research studies were the new mathematical modelling concept recently developed by the Principal Investigator, as well as a non-dimensionalization scheme which, among other things, served as a tool for comparison. The major goal of the second quarter was to research the possibility of obtaining closed-form solutions to the free vibrations problems of composite aircraft wings having a warping constraint and elastic coupling, a task that apparently had never been successfully accomplished previously. The accomplishment of this task is generally believed to be vital to the understanding of the dynamic or stability phenomena of such aircraft wings. Also included in the objectives for the second quarter were (a) a preliminary investigation of the effects of chordwise bending on the vibrations of orthotropic plates; (b) examination of

interdependency of design variables in tailoring technology, and (c) the formulation of explanations for some startling new results obtained during the first quarter.

The goals of the third and fourth quarters were (a) to refine the newly discovered closed-form solution approach in order to use it to study the constrained warping phenomena at higher vibrational frequencies, (b) to assess the implications and impact of what has been accomplished so far on the direction of this research program, and, (c) to prepare the AIAA Paper No. 86-1006, a paper based on the preliminary results of this research program, which was presented at the AIAA/ASME/ASCE/AHS 27th Structures, Structural Dynamics and Materials Conference, held in San Antonio, Texas, May 19-21, 1986.

## 5.0 STATUS OF RESEARCH EFFORT

### 5.1 INTRODUCTION

During the first year, the research program progressed according to plan resulting in the accomplishment of the goals defined for the period. These goals, which have been defined in Section 2 of this report, basically include a preliminary formulation and investigation of the aeroelasticity of supermaneuverable type aircraft wings (fabricated of composite materials) in order to determine an effective and efficient means with which to accurately estimate and study the effects of the "restraint of warping" phenomenon on their vibration and mechanism of aeroelastic instability.

In the first quarter, during the preliminary study, Dr. G.A. Oyibo investigated the free vibrations and the aeroelastic flutter aspect of this phenomenon, using an affine transformation concept and Professor T.A. Weisshaar studied the aeroelastic divergence aspect (of the phenomenon), using a non-dimensionalization scheme. The results obtained seem to indicate that although three different problems were investigated (using two different approaches) there were some general phenomenological trends that appeared to be common to all three of them. Summarized versions of these investigations were presented in the first quarterly report. Journal articles based on the results of these investigations are being prepared to be submitted to the AIAA Journal of Aircraft for publication.

In the beginning of the second quarter, Dr. Oyibo was able to formulate an explanation for the startling new results obtained during the first quarter. During the second quarter Dr. Oyibo was also able to establish the existence of closed-form solutions for the free vibrations of composite aircraft wings with the constraint of warping and elastic coupling. The preliminary results, which

appear to be exciting, confirm some known trends and expose some new ones. Professor T.A. Weisshaar examined the interdependence between some design variables employed in the tailoring technology as well as the effects of chordwise bending on the frequencies of orthotropic plates.

A more in-depth investigation of the newly discovered closed-form solutions was carried out during the third and fourth quarters to further investigate the dynamic phenomena of composite aircraft wings with the constraint of warping and elastic coupling.

In particular, higher vibrational frequencies are being generated and studied carefully to determine trends. An assessment of some of the impacts and implications of results obtained in the research thus far reveals that (a) the newly discovered closed-form approach may be extendable to study the stability problems and (b) it now appears possible (and certainly desirable) to develop a numerical superelement that can capture the essence of the warping phenomena for studying more irregular wing configurations. During the third quarter, also, AIAA Paper No. 86-1006, based on the preliminary results of this research program, and accepted to be presented at the AIAA/ASME/ASCE/AHS 27th Structures, Structural Dynamics and Materials Conference in San Antonio, Texas, May 19-21, 1986 was prepared, and subsequently presented at the conference.

## 5.2 ACCURATE DYNAMIC THEORY FOR SUPERMANEUVERABLE AIRCRAFT WINGS

by Gabriel A. Oyibo

### 5.2.1 Introduction

Modern supermaneuverable aircraft concepts benefit a great deal from, among other things, significant advances in materials technology and the availability of more accurate aerodynamic prediction capabilities. Supermaneuverability as a design goal invariably calls for an optimization of the design parameters. Optimization may be partially accomplished for example, by using composite materials to minimize weight. Indeed, it has been known that these composite materials can be tailored properly to resolve the dynamic or static instability problems of these types of aircraft. The concept is referred to as aeroelastic tailoring.

While aeroelastic tailoring has tremendous advantages in the design of an aircraft, the analysis which provides the basis for the aeroelastic tailoring itself is generally very involved. This is rather unfortunate since a good fundamental physical insight of the tailoring mechanism is required for accurate and reliable results.

In this investigation an attempt is made to look at some dynamics theories that can be used to understand the aeroelastic tailoring mechanism. Specifically, the accuracy of the St. Venant torsion theory which is relatively simple and frequently used in aeroelastic analysis is examined with particular reference to the effects of the wings aspect ratio as well as other design parameters. An accurate torsion/twist theory is particularly significant for supermaneuverable aircraft wings since supermaneuverability is basically characterized by high angle of attack.

Although earlier studies(1,2,3) have indicated that the St. Venant's torsion theory is reasonably accurate except for aircraft wings with fairly low aspect ratios, the theory supporting that conclusion was based on the assumption that the wing is constructed of isotropic materials. Basically the St. Venant's torsion theory assumes that the rate of change of the wing's twist angle with respect to the spanwise axis is constant. This assumption is hardly accurate particularly for modern aircraft construction in which different construction materials are employed and the aerodynamic loads vary significantly along the wing's span. However, References 1-3 have shown that (in spite of such an inaccurate assumption) the main parameter that determines the accuracy of the St. Venant's theory is the wing's aspect ratio. Thus, it was determined that the theory is fairly accurate for moderate to high aspect ratio wings constructed of isotropic materials. In recent studies (4,5,6) however, it has been shown that for wings constructed of orthotropic composite materials, the conclusion of References 1-3 need to be modified. Rather than using the geometric aspect ratio of the wing to determine the accuracy of St. Venant's twist theory, it was suggested that a generic stiffness ratio as well as an effective aspect ratio which considers the wing's geometry and the ratio of the principal directional stiffness should be considered in establishing the accuracy of St. Venant's theory.

The present investigation is related to the studies that were initiated in References 5 and 6. In this study the first task was to examine the role of coupling (both mass and elastic coupling) on the accuracy of St. Venant's theory applied to static problems. It was discovered that coupling plays a very significant role on the accuracy of St. Venant's twist theory. The second task was to investigate the torsional vibration for a flat plate model of an aircraft wing fabricated of composite materials in which the constrained warping

phenomenon is more realistically represented, with particular emphasis on higher frequencies and to compare results with those from a representation based on St. Venant's theory.

### 5.2.2 Formulation

Consider an aircraft wing fabricated of composite materials and mathematically idealized as a cantilevered plate subjected to an aerodynamic flow over its surfaces. The mathematical statement of the virtual work theorem for such a plate model is well known and documented. It is also known that such mathematical statements of the virtual work theorem for a laminated plate model are characterized with the existence of so many variables (in the statement), reflecting the various directional properties for the laminated plate model, which would tend to interfere with any physical insight that might be desired from a phenomenological analysis employing such a mathematical statement. The newly discovered affine transformation concept (5,6 and 7) was developed principally to resolve such a problem.

This new concept therefore can be used to evolve the mathematical statement of the virtual work theorem in an affine space given by the following equation.

$$\begin{aligned} \delta \bar{U}_0 = 0 = & 2 \int_0^t \iint_A \left\{ (w, x_0 x_0)^2 + 2D^* \left[ (1 - \epsilon)(w, x_0 y_0)^2 \right. \right. \\ & \left. \left. + \epsilon w, x_0 x_0 w, y_0 y_0 \right] + (w, y_0 y_0)^2 + L_1 w, x_0 x_0 w, x_0 y_0 + L_2 w, y_0 y_0 w, x_0 y_0 \right\} dx_0 dy_0 dt \\ & - \frac{\delta}{2} \int_0^t \iint_A \rho_0 \dot{w}^2 dx_0 dy_0 dt + \int_0^t \iint_A \Delta p_0 \delta w dx_0 dy_0 dt \end{aligned} \quad (1)$$

where:

$$\begin{aligned} \bar{U}_0 &= \frac{\bar{U}}{D_{22}} \left( \frac{D_{22}}{D_{11}} \right)^{1/4} ; \quad D^* = \frac{D_{12} + 2D_{66}}{(D_{11}D_{22})^{1/2}} ; \quad \epsilon D^* = \frac{D_{12}}{(D_{11}D_{22})^{1/2}} \\ L_1 &= \frac{4D_{16}}{(D_{11})^{3/4}(D_{22})^{1/4}} ; \quad L_2 = \frac{4D_{26}}{(D_{11})^{1/4}(D_{22})^{3/4}} \quad (2) \\ \Delta^0_0 &= \frac{\Delta^0}{D_{22}} ; \quad \rho_0 = \frac{\bar{\rho} h}{D_{22}} \end{aligned}$$

$D_{ij}$  are the elastic constants,  $\rho$ , is the material density,  $\Delta p$  is the differential pressure distribution,  $w$  is the displacement,  $t$  is the time,  $A$  integrals represent area integrals and  $\bar{h}$  is the wing box depth.

Equations (1) and (2) therefore form the basis of the newly developed methodology. The equations of motion of a plate model of an aircraft wing can now be derived by prescribing a realistic wing displacement and using Equation (1).

When Equation (1) is compared to its physical space counterpart, it is seen that Equation (1) has fewer variables. It is also seen that Equation (1) contains only non-dimensionalized stiffness quantities (compared to dimensional stiffness quantities in its physical space counterpart). Another feature of

this new methodology which makes it unique is that the non-dimensionalization (a consequence of the affine transformation) is accomplished before assuming the wing deformations. This means that the non-dimensionalization is independent of how the wing deforms. A non-dimensionalization scheme that depends on a particular assumption of the wings deformations could lead the analyst to an incorrect physical interpretation of results, since the wing's deformations assumptions have inherent errors because they are based on the analyst's judgments and experience. This observation may become clearer during the evolution of a warping parameter with which to study the effects of the warping constraint phenomenon on the status and dynamics of a wing fabricated of composite materials later in this study.

If the chordwise curvature is neglected in an initial approximation, the wing's deformation may be assumed as follows:

$$w(t, x_0, y_0) = h_0(t, y_0) + x_0 \alpha_0(t, y_0) \quad (3)$$

where  $h_0$  and  $\alpha_0$  are the bending and twisting displacements respectively.

It can be shown that when Equation (3) is substituted into Equation (1) and the variational calculus is carried out for arbitrary  $h_0$  and  $\alpha_0$ , the following equations of motion are obtained:

$$\begin{aligned} a_1 h_0^{iv} + a_2 \alpha_0^{iv} + a_5 \alpha_0^{iii} + \rho_0 a_1 \ddot{h}_0 + \rho_0 a_2 \ddot{\alpha}_0 &= L_0 \\ a_2 h_0^{iv} - a_5 h_0^{iii} + a_3 \alpha_0^{iv} - a_4 \alpha_0'' + \rho_0 a_3 \ddot{\alpha}_0 + \rho_0 a_2 \ddot{h}_0 &= M_0 \end{aligned} \quad (4)$$

where:

$$a_1 = \int \frac{\bar{c}_0}{e\bar{c}_0} dx_0 \quad ; \quad a_2 = \int \frac{\bar{c}_0}{e\bar{c}_0} x_0 dx_0$$

$$a_3 = \int \frac{\bar{c}_0}{e\bar{c}_0} x_0^2 dx_0 \quad ; \quad a_4 = 2 \int \frac{\bar{c}_0}{e\bar{c}_0} \theta^*(1-\epsilon) dx_0 \quad (5)$$

$$L_0 = \int \frac{\bar{c}_0}{e\bar{c}_0} \Delta p_0 dx_0 \quad ; \quad a_5 = \int \frac{\bar{c}_0}{e\bar{c}_0} L_2 dx_0$$

$$M_0 = \int \frac{\bar{c}_0}{e\bar{c}_0} x_0 \Delta p_0 dx_0$$

$$-\infty < e < 0 \quad ; \quad \bar{c}_0 = \frac{c_0}{1-e}$$

$$(\quad)' = \frac{\partial}{\partial y_0}, (\dot{\quad}) = \frac{\partial}{\partial t} \quad (6)$$

### 5.2.3 Evolution of Warping Parameters

The evolution of the warping parameter with which to study the aeroelastic warping constraint phenomenon for wings fabricated of composite materials is a process that depends on the sophistication of the wing's mathematical model; whether coupling effects are included, whether the wing's chordwise curvatures are included and so on. Therefore, any warping parameter is as good as the corresponding wing's displacements assumptions. However, Equation (1) makes it possible for the analyst to determine its effective independent variables even before the displacement assumptions are made.

By non-dimensionalizing the spanwise space variable in Equation (4), depending on whether one is interested in the static, dynamic, coupled or uncoupled displacements, one of the following warping parameters may be useful.

$$\lambda_c = \frac{\ell_o}{c_o} \sqrt{\frac{3}{2} D_o^*} \quad (7)$$

$$\bar{\lambda}_c = \frac{\ell_o}{c_o} \sqrt{\frac{3}{2} (D_o^* - \frac{L^2}{8})} \quad (8)$$

$$\bar{\lambda}_c = \frac{\ell_o}{c_o} \sqrt{\frac{3}{2} \frac{D_o^*}{1-3\bar{a}_2^2}} \quad (9)$$

where

$$D_o^* = D^* (1 - \epsilon) \quad ; \quad \bar{a}_2 = \frac{a_2}{c_o} \quad (10)$$

$(\ell_o/c_o)$  is defined as the wing's effective aspect ratio and  $D_o^*$  and  $L$  are the generalized stiffness and coupling ratios respectively (defined in earlier work such as References 5 and 6).

Equations (7) thru (9) represent the appropriate warping parameter for dynamic deformation, static displacement with elastic cross-coupling, and static deformation with "geometric" coupling ( $\epsilon \neq 1$ ) respectively.

It was discovered in this study that evolving the warping parameter in a manner shown in Equations (1) thru (3), should enable one to investigate the effects

of warping on the composite wing's dynamics (or the accuracy of St. Venant's theory) effectively. From the lamination theory for composites it is known that while  $D_{0*}$  and  $(\ell_0/c_0)$  are always positive,  $L$  and  $\bar{a}_2$  can be positive or negative. However, from Equations (1) and (3), it is clear that whether a composite wing has positive or negative coupling, the warping effect (in terms of  $\bar{\lambda}_c$ ) is unchanged.

#### 5.2.4 Results and Conclusions

By using the evolved warping parameters defined in Equations (7) and (8) and the appropriate boundary conditions, the boundary value or the eigenvalue problems associated with Equation 4 are solved in a closed-form manner to determine the wing's static twist and torsional vibrations respectively.

The results are shown in Figures 1 thru 10. Figure 1 shows a comparison of the static wing tip twist obtained in the present study and that obtained via St. Venant's twist theory in the presence of statically distributed aerodynamic forces and low to moderate coupling. Figures 2 thru 5 show the trend for concentrated aerodynamic forces and substantial coupling. In Figures 1 thru 5 it is seen that the presence of coupling makes the errors of St. Venant's theory worse. This seems to suggest that the more sophisticated theory is more important for wings with coupling (e.g., wings aeroelastically-tailored using elastic cross-coupling).

Figures 2 thru 5 also show that nonconservative errors  $\left(\left|\frac{\alpha^{(1)}}{\alpha_{STV}^{(1)}}\right| > 1\right)$  are possible. In Figures 6 thru 10 the results of the torsional vibrations are shown for a wing without coupling. Figure 6 is particularly striking by showing clearly that the errors of St. Venant's theory are more significant for higher modes of torsional vibrations. While the errors could be bad enough for the first mode

for  $\lambda_c < 4$ , it is encouraging to find that errors of the lower modes are better than those for higher modes since the critical aeroelastic instabilities generally occur at lower frequencies. In Figures 7 thru 10 the frequency parameters are plotted against the effective aspect ratio of the wing for each  $\bar{D}_0$  for the first four modes. The St. Venant's results should be recovered for very large effective aspect ratio.

Using Figures 1 thru 10, the following conclusions can be summarized: (i) ignoring warping arbitrarily using St. Venant's theory could result in very significant errors (as high as over 80% errors) in analytical results for composite aircraft wings, (ii) warping is more important (St. Venant's theory is less accurate) for wings with coupling, (iii) St. Venant's theory (which has always been shown to be conservative (1,2)), can be non-conservative or St. Venant's approximation can lead to an unsafe design error (under design rather than over design from a stability point of view), and (iv) warping is more important (St. Venant's theory is less accurate) for higher vibration modes.

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# WING TIP TWIST RATIOS FOR SIMPLE AND MORE INVOLVED THEORY (DISTRIBUTED AERODYNAMIC LOAD AND LOW TO MODERATE COUPLING)

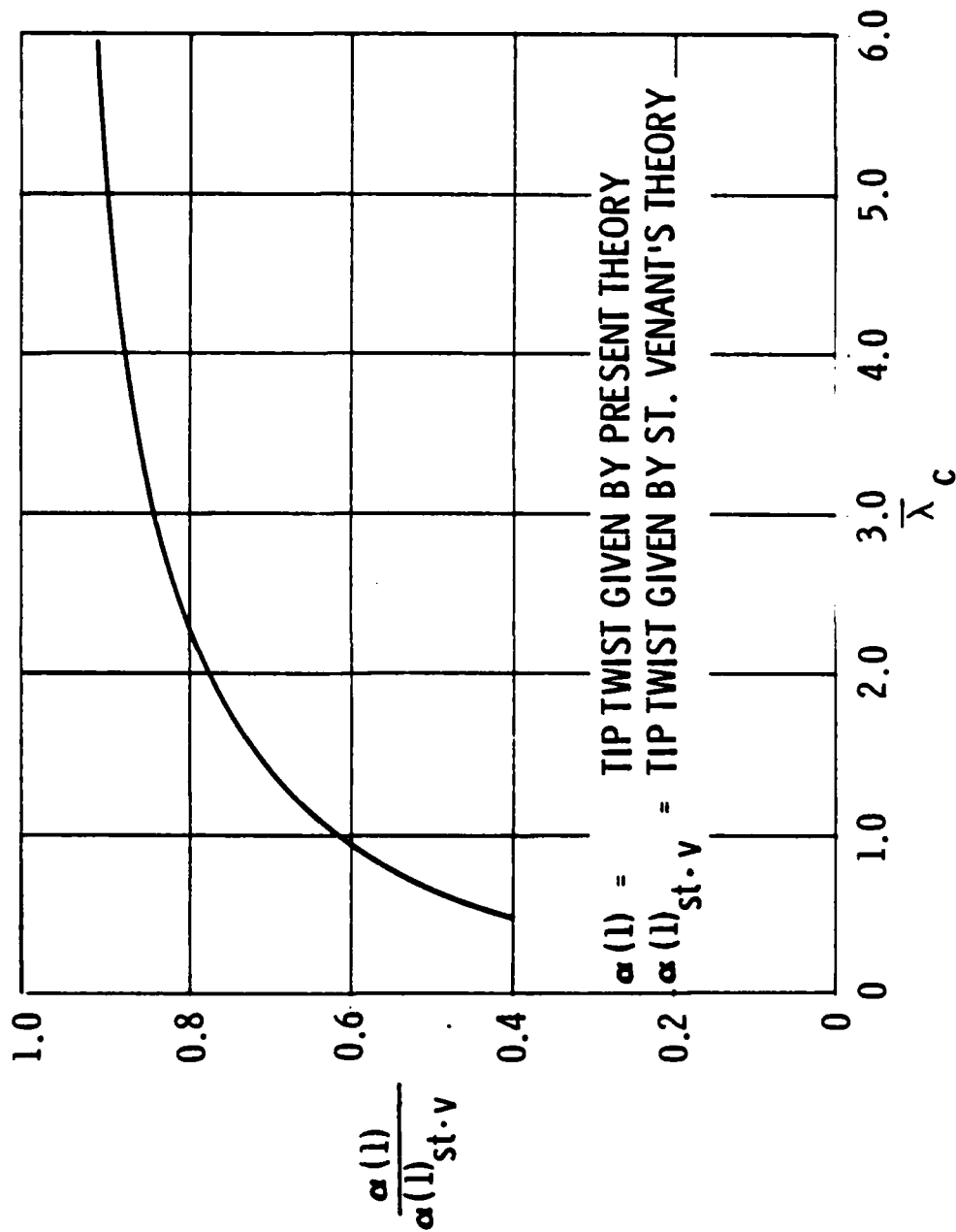


Figure 1

# WING TIP TWIST RATIOS COMPARING SIMPLE AND MORE INVOLVED THEORY (WITH SUBSTANTIAL COUPLING)

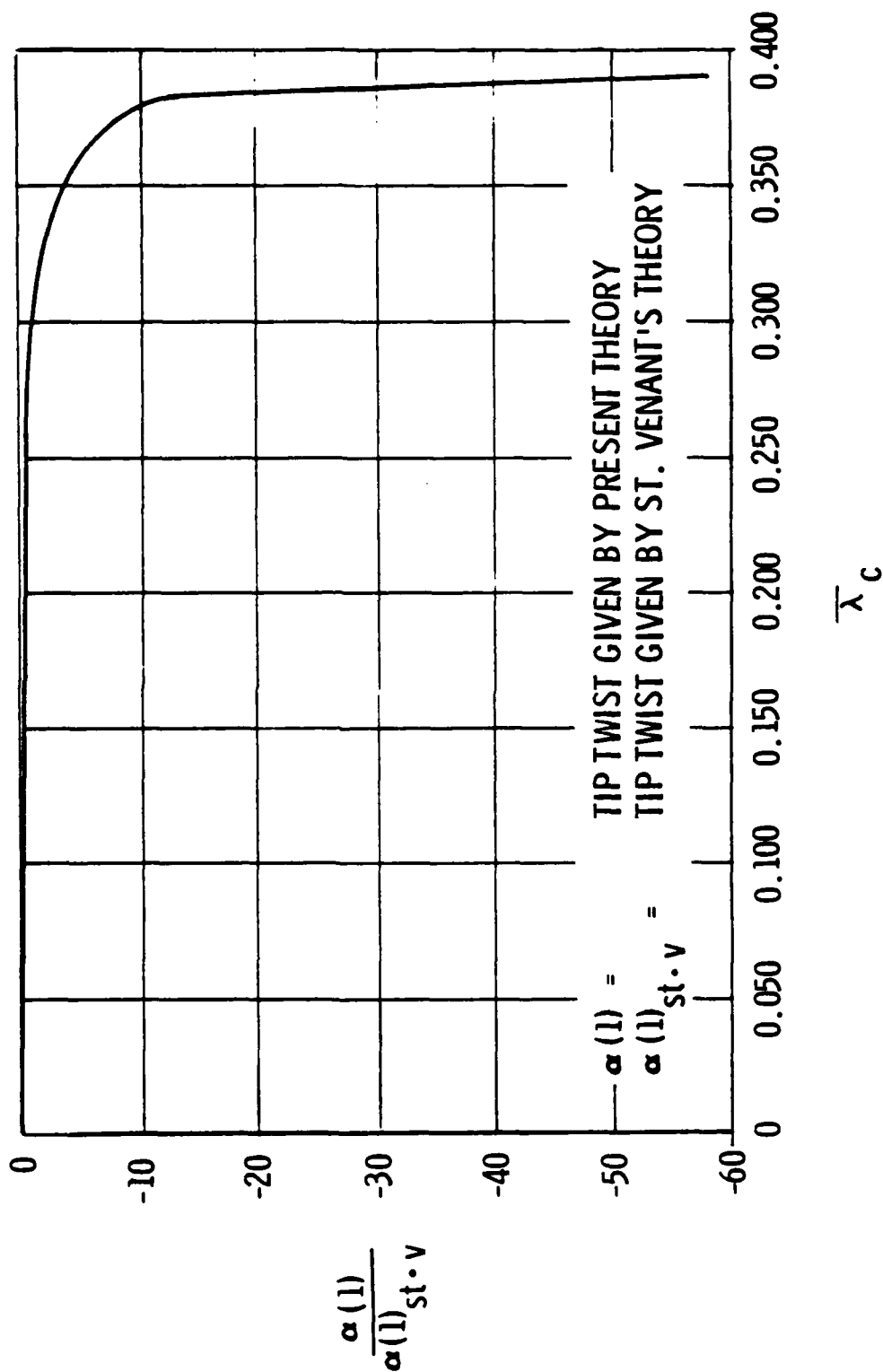


Figure 2

# WING TIP TWIST RATIOS COMPARING SIMPLE AND MORE INVOLVED THEORY (WITH SUBSTANTIAL COUPLING)

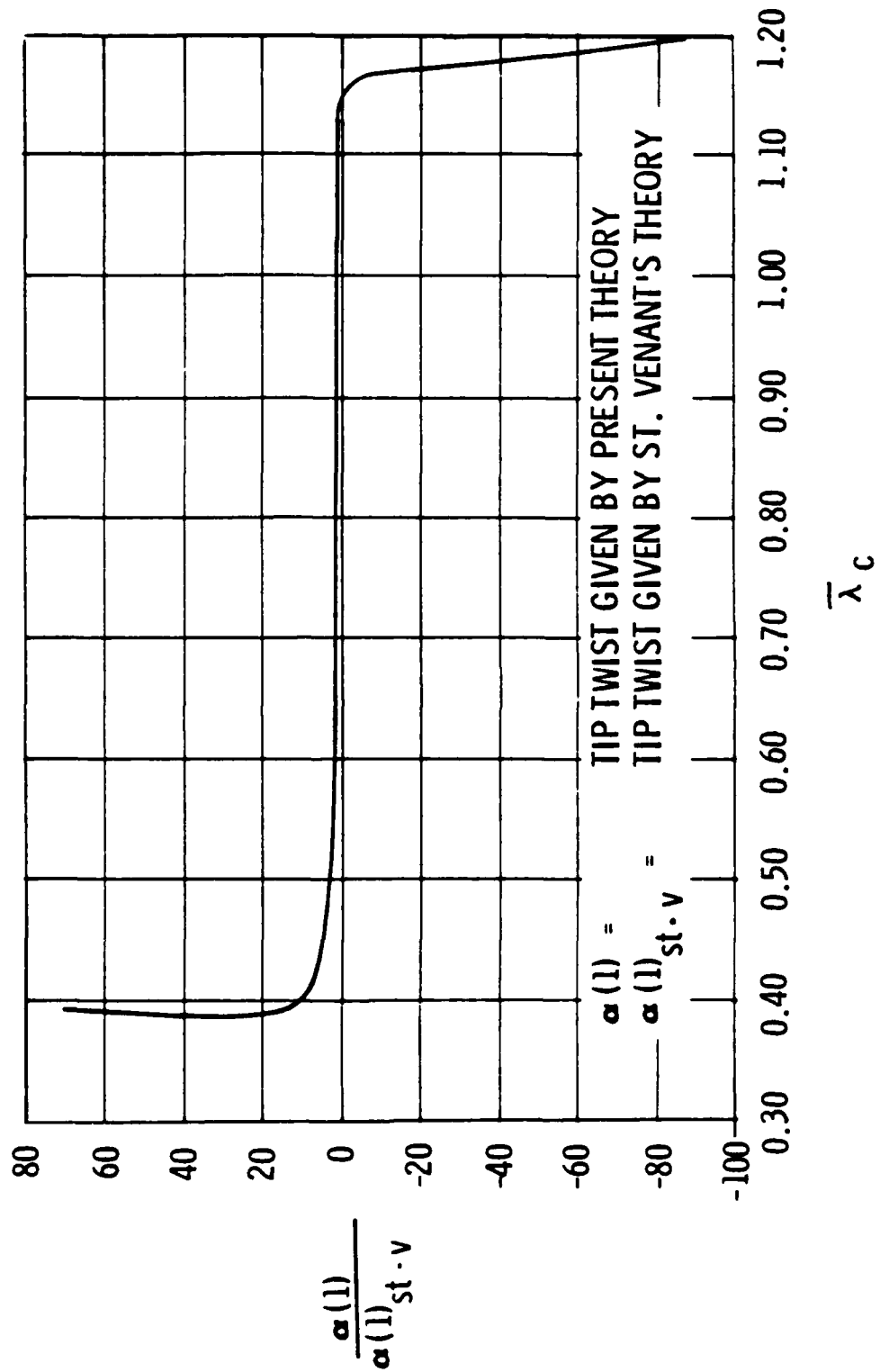


Figure 3

# WING TIP TWIST RATIOS COMPARING SIMPLE AND MORE INVOLVED THEORY (WITH SUBSTANTIAL COUPLING)

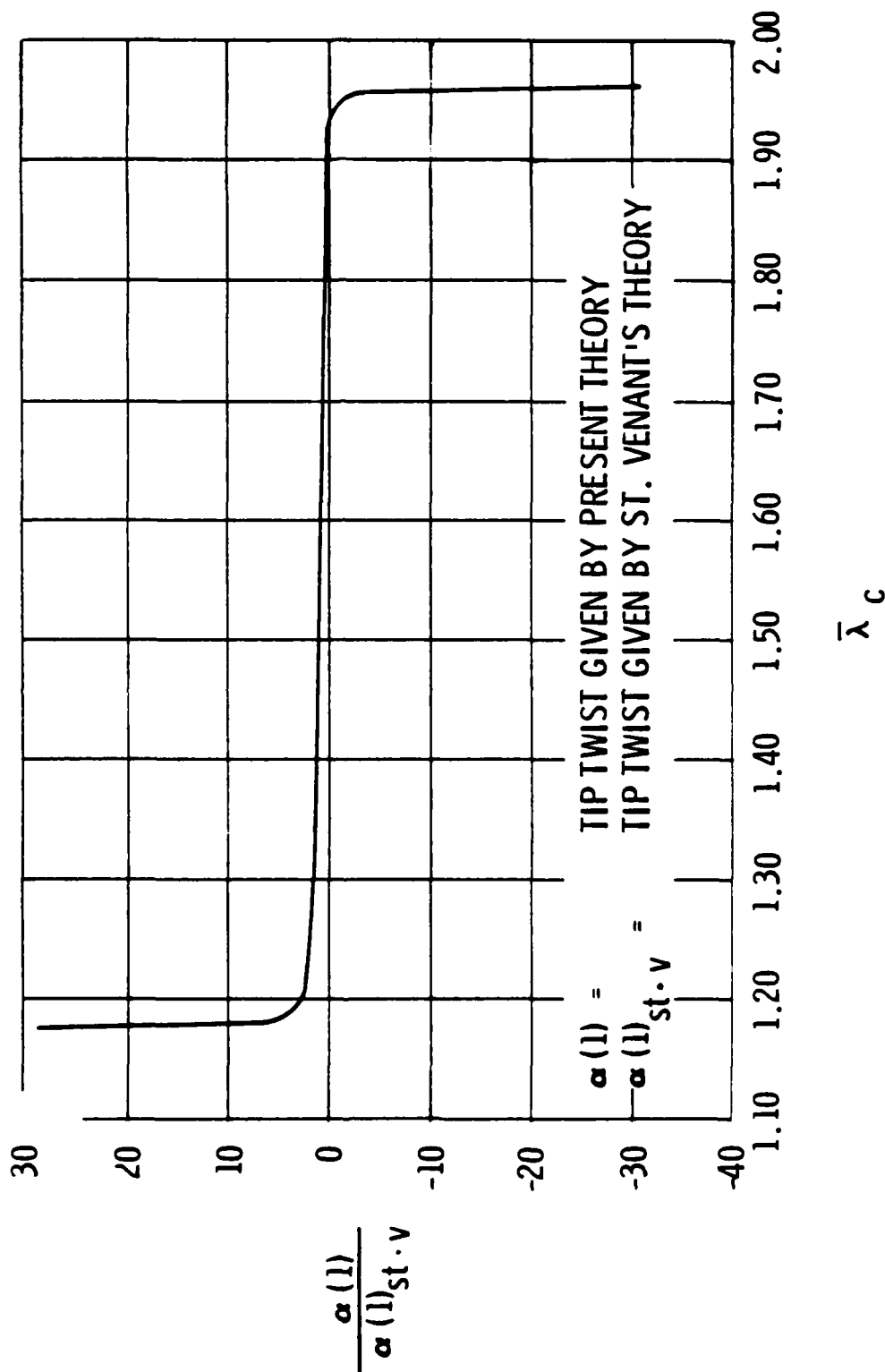


Figure 4

# WING TIP TWIST RATIOS COMPARING SIMPLE AND MORE INVOLVED THEORY (WITH SUBSTANTIAL COUPLING)

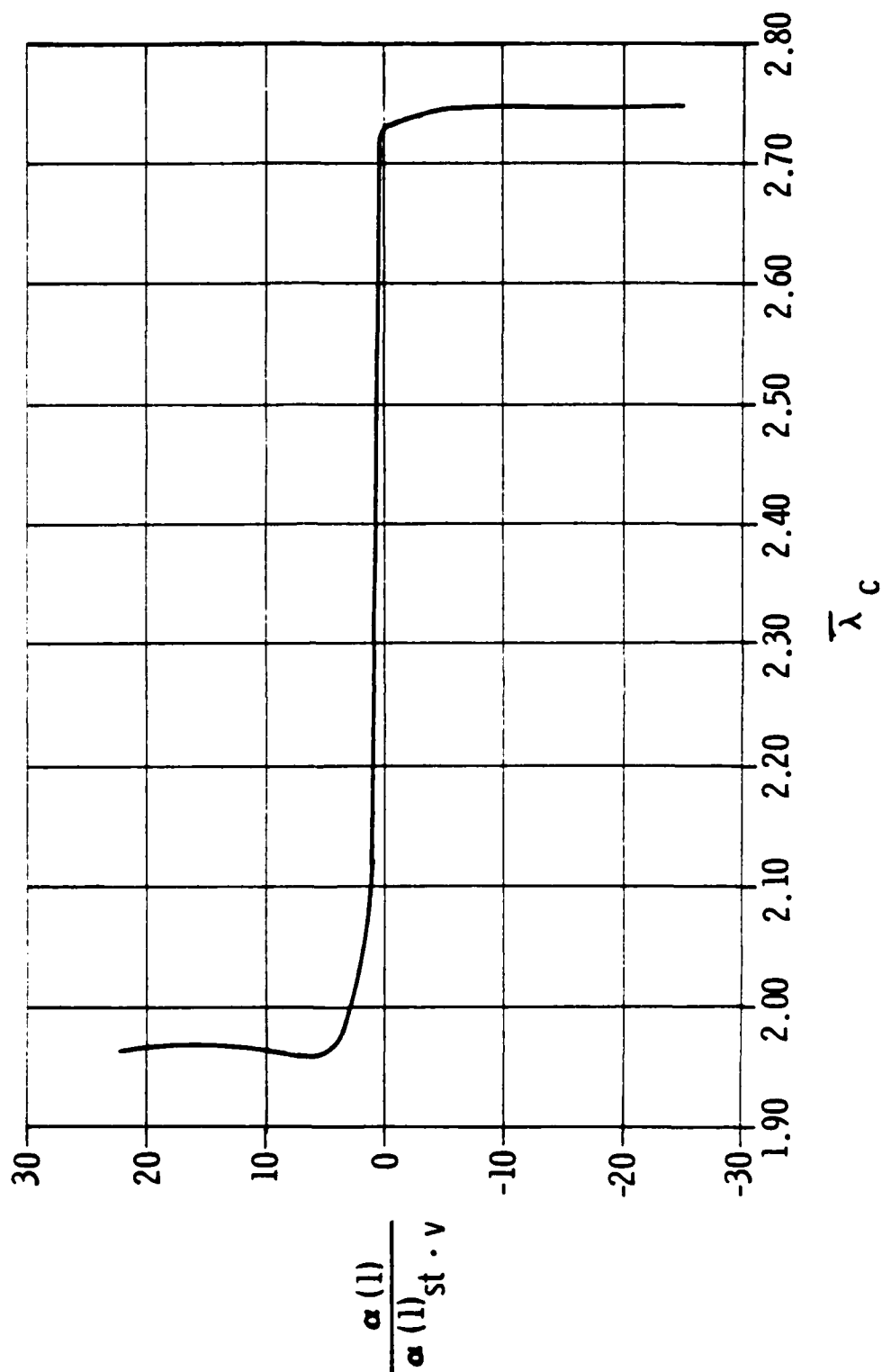


Figure 5

# 1st TORSIONAL FREQUENCY PARAMETER USING A MORE INVOLVED THEORY

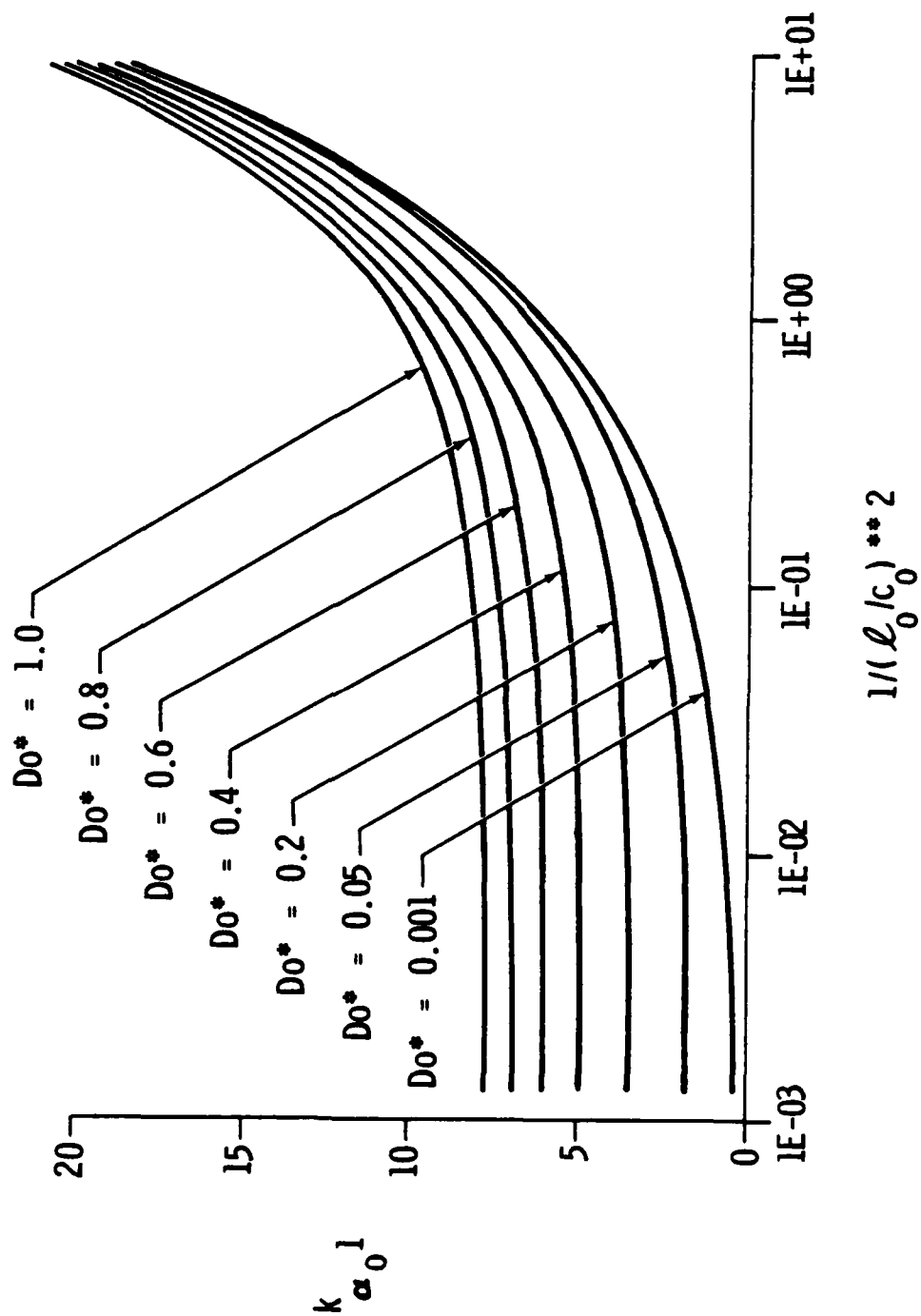


Figure 6

# 2nd TORSIONAL FREQUENCY PARAMETER USING A MORE INVOLVED THEORY

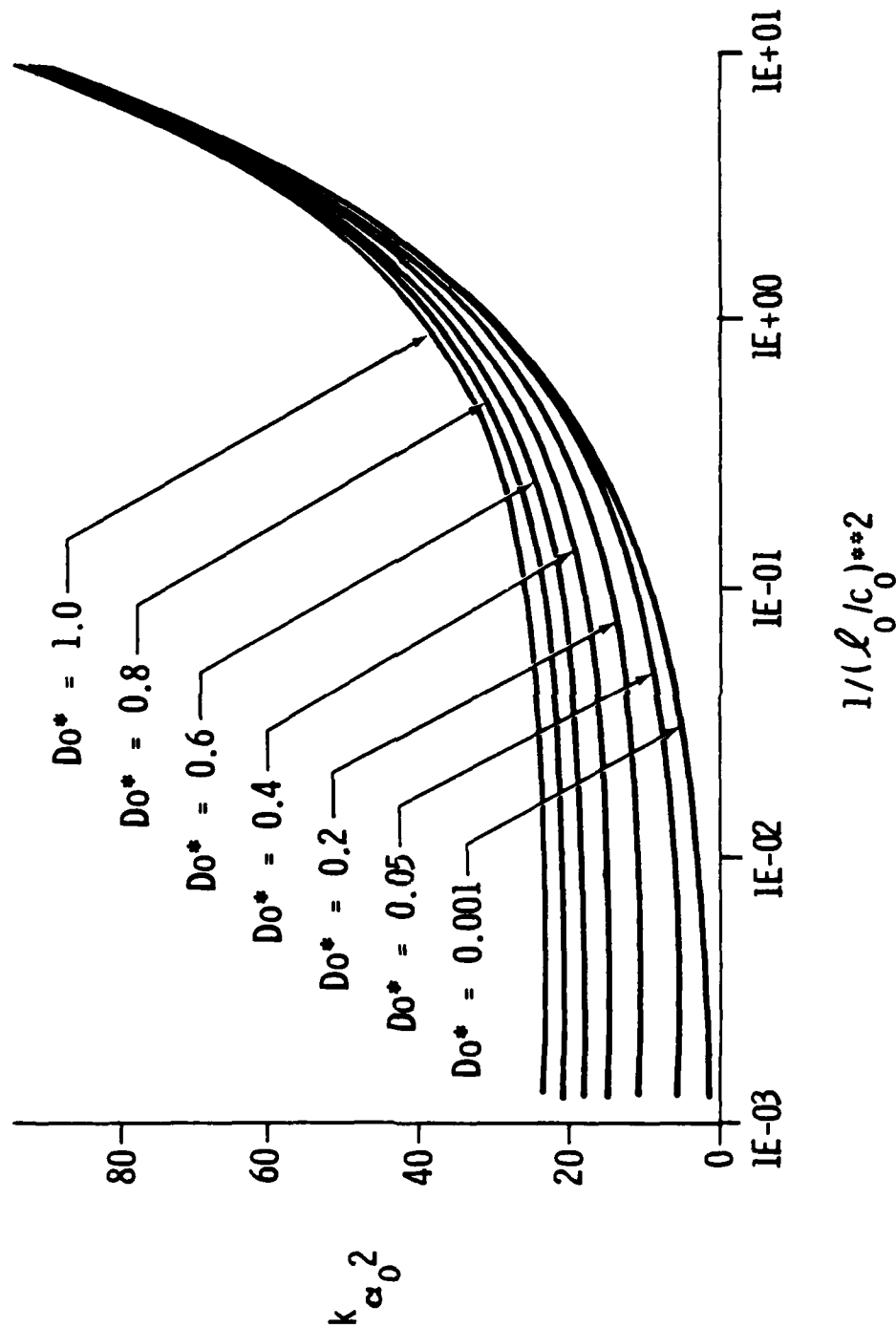


Figure 7

# 3rd TORSIONAL FREQUENCY PARAMETER USING A MORE INVOLVED THEORY

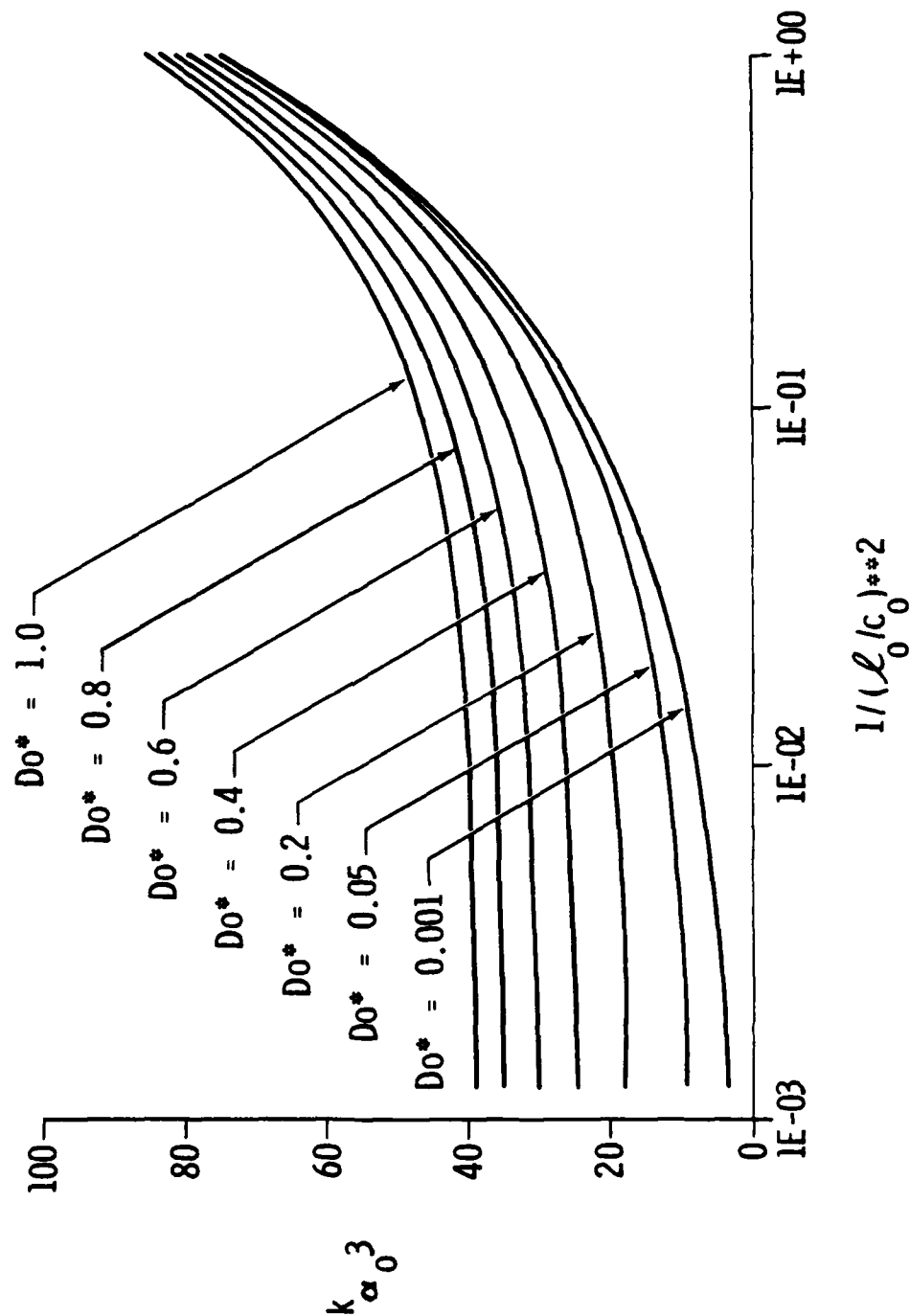


Figure 8

# 4th TORSIONAL FREQUENCY PARAMETER USING A MORE INVOLVED THEORY

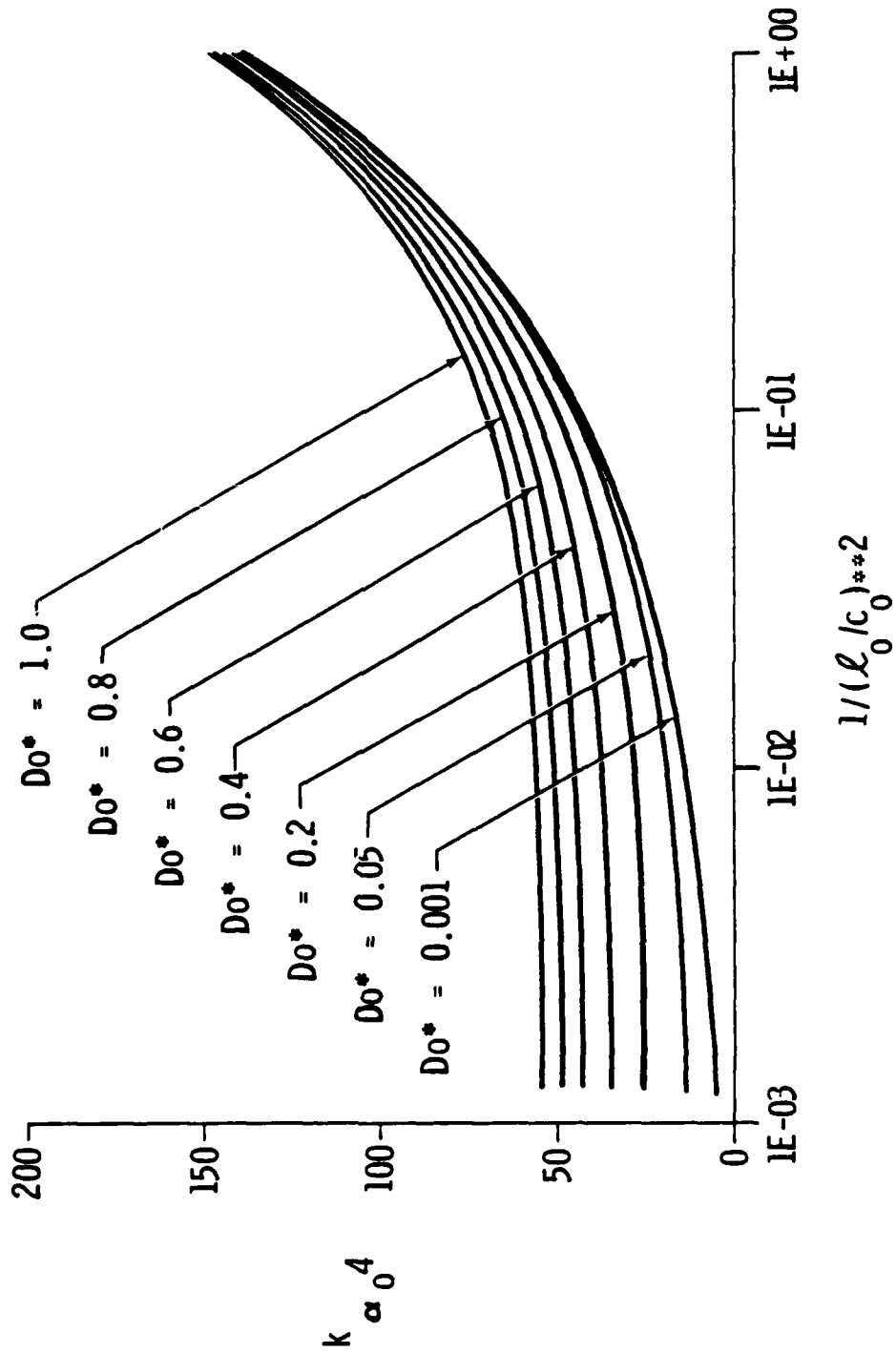


Figure 9

### 5.3 AEROELASTIC FLUTTER PHENOMENON FOR COMPOSITE AIRCRAFT WINGS WITH WARPING CONSTRAINT by Gabriel A. Oyibo

#### 5.3.1 Introduction

As one goes from aeroelastic research and development to applications, it is absolutely important that one clearly understands the limitation of the theoretical idealizations employed during the research and development stage. Such theoretical idealizations may have intricate practical implications, particularly for aircraft lifting surfaces which are fabricated of composite materials, due to the inherent mathematical and physical understanding difficulties generally encountered in the analysis of composite systems.

The significance of the above mentioned point may be evaluated in the following context: it has been established(1,2,3) that the effects of spanwise sectional warping constraints on the aeroelastic phenomena like the bending-torsion flutter may be ignored for a warping constrained cantilevered lifting surface provided that the surface has a high structural aspect ratio. A "freely warping" lifting surface in this context may be defined as one whose twist distribution varies linearly with its spanwise length (measured from its root). Therefore, since it is almost impossible in practice to construct aircraft wings with such a twist distribution, it is probably safe to assume that most wings have constrained warping. The idealization of wings as "freely warping", which simplifies aeroelastic analysis considerably, is based on the conclusions from investigations like those of References 1-3, in which it was shown that a high aspect ratio wing may be treated aeroelastically as "freely warping". However, as was pointed out in References 4 and 5, these conclusions are strictly true for lifting surfaces constructed of isotropic materials, and therefore should be modified (or generalized) for those fabricated of composite materials. The

suggested generalizations(4,5) were basically meant to caution analysts from using idealizations which were strictly meant for lifting surfaces fabricated of isotropic materials for those fabricated of composite materials.

The study of the significance of the warping constraint in the binary aero-elastic flutter phenomenon for orthotropic composite wings is the subject of this section. In Reference 6, the study of the binary flutter mechanism for orthotropic composite wings was carried out ignoring the warping constraint (permitting the use of St. Venant's twist theory). The results of the present study which considers the warping constraint in the analysis is eventually compared to those of Reference 6 and a set of interesting conclusions is drawn. Two-dimensional unsteady aerodynamics and an advanced composite lamination theory were used in both studies along with a new dynamic equivalence concept.(6) This new concept is based on the following newly defined generic quantities. Consider a composite wing with an effective aspect ratio,  $\ell_0/c_0$ , given by

$$\left(\frac{\ell_0}{c_0}\right) = \left(\frac{\ell}{c}\right) \left(\frac{D_{11}}{D_{22}}\right)^{\frac{1}{4}} \quad (1)$$

and generic stiffness ratios  $D^*$  and  $D_0^*$  given by,

$$D^* = \frac{D_{12} + 2D_{66}}{(D_{11}D_{22})^{1/2}} ; D_0^* = D^*(1 - \epsilon), \quad (2)$$

where  $\epsilon$  is a generalized Poisson's ratio effect.

### 5.3.2 Formulation

The traditional approach for formulating the equations of motion for flat plate models of aircraft wings fabricated of composite materials is well documented and known to lead to mathematical equations containing numerous variables with no known bounds. The resulting flutter analysis is generally characterized with results that are hardly explicable. The new methodology, which provides

solutions to such problems, employs an affine transformation concept to define equivalent dynamic systems with which to investigate the mechanisms of flutter. Such dynamically equivalent systems automatically provide easy means for designing wind tunnel test models which are accurate. The new methodology basically transforms the mathematical statement of the virtual work theorem for a laminated plate model of an aircraft wing, undergoing aeroelastic oscillations, into a more convenient counterpart in an affine space, based on Figure 1, given by the following equation.

$$\begin{aligned} \delta \bar{U}_0 = 0 = & \frac{\delta}{2} \int_0^t \iint_A \left\{ (w_{,x_0 x_0})^2 + \right. \\ & 2D^* \left[ (1 - \epsilon)(w_{,x_0 y_0})^2 + \epsilon w_{,x_0 x_0} w_{,y_0 y_0} \right] + (w_{,y_0 y_0})^2 + \\ & \left. L_1 w_{,x_0 x_0} w_{,x_0 y_0} + L_2 w_{,y_0 y_0} w_{,x_0 y_0} \right\} dx_0 dy_0 dt - \\ & \frac{\delta}{2} \int_0^t \iint_A \rho_0 \dot{w}^2 dx_0 dy_0 dt + \int_0^t \iint_A \Delta p_0 \delta w dx_0 dy_0 dt \end{aligned} \quad (3)$$

where,

$$\begin{aligned}\bar{U}_0 &= \frac{\bar{U}}{D_{22}} \left( \frac{D_{22}}{D_{11}} \right)^{1/4} \\ D^* &= \frac{D_{12} + 2D_{66}}{(D_{11}D_{22})^{1/2}} \\ \epsilon D^* &= \frac{D_{12}}{(D_{11}D_{22})^{1/2}} ; \quad D_0^* = D^*(1 - \epsilon) \end{aligned} \quad (4)$$

$$\begin{aligned}L_1 &= \frac{4D_{16}}{(D_{11})^{3/4}(D_{22})^{1/4}} ; \quad L_2 = \frac{4D_{26}}{(D_{11})^{1/4}(D_{22})^{3/4}} \\ \Delta p_0 &= \frac{\Delta p}{D_{22}} ; \quad \rho_0 = \frac{\bar{\rho} h}{D_{22}}\end{aligned}$$

$D_{ij}$  are the elastic constants,  $\rho$ , is the material density,  $\Delta p$  is the differential pressure distribution,  $w$  is the displacement,  $t$  is the time,  $A$  integrals represent area integrals and  $h$  is the wing box depth.

Equations (3) and (4) represent the basis for the new methodology. By prescribing realistic aircraft wing deformations, these equations can be used to derive equations of motion.

### 5.3.3 New Similarity Theory

Since the equations of motion for the aircraft wing are derivable from the virtual work theorem given by Equation (3), it is now clear (even before the

actual equations of motion are derived) that the parameters, whose similarities must be preserved between the real aircraft wing and its model are those present in Equation (3). Therefore, rather than trying to preserve the similarities of  $D_{ij}$ , as well as the fiber orientation angles as may be necessary in Equation (1), Equation (3) makes it possible to require only the preservation of the similarities of  $D_0^*$ ,  $(\frac{D_{11}}{D_{22}})$ ,  $L_1$ , and  $L_2$  over and above the requirements for aircraft wings fabricated of isotropic materials. In what follows a specific displacement assumption will be used to illustrate this new concept.

Consider the following bending-torsion displacement assumption:

$$w(t, x_0, y_0) = h_0(t, y_0) + x_0 \alpha_0(t, y_0) \quad (5)$$

where

$h_0$  and  $\alpha_0$  are the bending and torsional displacements respectively.

When Equation (5) is substituted into Equation (3) the following equations of motion for the wing can be derived with the help of variational calculus.

$$\begin{aligned} a_1 h_0^{iv} + a_2 \alpha_0^{iv} + a_5 \alpha_0^{iii} + \rho_0 a_1 \ddot{h}_0 + \rho_0 a_2 \ddot{\alpha}_0 &= L_0 \\ a_2 h_0^{iv} + a_3 \alpha_0^{iv} - a_5 h_0^{iii} - a_4 \alpha_0^{'''} + \rho_0 a_3 \ddot{h}_0 + \rho_0 a_2 \ddot{\alpha}_0 &= M_0 \end{aligned} \quad (6)$$

where

$$\begin{aligned} a_1 &= \int_{-c_0/2}^{c_0/2} dx_0 & a_2 &= \int_{-c_0/2}^{c_0/2} x_0 dx_0 \\ a_3 &= \int_{-c_0/2}^{c_0/2} dx_0 & a_4 &= 2 \int_{-c_0/2}^{c_0/2} D^*(1-\epsilon) dx_0 \\ a_5 &= \int_{-c_0/2}^{c_0/2} L_2 dx_0 & L_0 &= \int_{-c_0/2}^{c_0/2} \Delta \rho_0 dx_0 & M_0 &= \int_{-c_0/2}^{c_0/2} \Delta \rho_0 x_0 dx_0 \end{aligned} \quad (7)$$

$$(\quad)' = \frac{\partial}{\partial y_0} (\quad) \quad (\dot{\quad}) = \frac{\partial}{\partial t} (\quad)$$

Equations (6) therefore give the coupled equations of motion for a flat plate model of an aircraft wing, fabricated of composite materials undergoing aeroelastic oscillations. The presence of  $a_3$  in Equation (6) indicates a realistic representation of aeroelastic oscillations with constrained warping. St. Venant's theory which ignores  $a_3$  will be compared to the present theory in the flutter analysis that follows.

#### 5.3.4 Unsteady Aerodynamics

In order to solve Equations (6) to determine aeroelastic instability boundaries, the aerodynamic forces and moments  $L_0$  and  $M_0$  would have to be generated by solving the aerodynamic equations. In the affine space, the aerodynamic equations are slightly more difficult to solve than in the physical space. However, the overall relative simplicity of the aeroelastic equations of motion certainly justifies the extra effort needed. Two-dimensional linear approximations and a Theodorsen type approach can be used to generate the incompressible harmonic unsteady aerodynamics used in this analysis. It can be shown that:

$$\begin{aligned}
 L_0 &= \frac{\pi}{r} \bar{\rho} c_o'^3 \omega^2 \left\{ L_{h_0} \bar{h}_0 + \left[ L_{\alpha_0} - L_{h_0} \left( e + \frac{1}{2} \right) \right] \alpha_0 \right\} \\
 M_0 &= \frac{\pi}{r} \bar{\rho} c_o'^4 \omega^2 \left\{ \left[ \frac{1}{2} - L_{h_0} \left( e + \frac{1}{2} \right) \right] \bar{h}_0 + \left[ M_{\alpha_0} \right. \right. \\
 &\quad \left. \left. + L_{h_0} \left( e + \frac{1}{2} \right)^2 - \left( L_{\alpha_0} + \frac{1}{2} \right) \left( e + \frac{1}{2} \right) \right] \alpha_0 \right\}
 \end{aligned} \tag{8}$$

$$\bar{\rho} = \frac{\rho_{\infty}}{D_{22}}$$

$$r = \left( \frac{D_{22}}{D_{11}} \right)^{1/4}$$

$$\bar{h}_0 = \frac{h_0}{c'_0} \quad (9)$$

$$(\bar{h}_0, \alpha_0) = (H_0, \alpha_0) e^{i\omega t}$$

$\rho$  is the air density,  $(H_0, \alpha_0)$  are the vibration amplitudes,  $\omega$  is the vibration frequency and  $e$  measures the location of the elastic axis relative to the mid-chord.

$L_{h_0}$  is the running aerodynamic lift coefficient due to the wing's bending vibrations in affine space,  $L_{\alpha_0}$  is the running aerodynamic lift coefficient due to the wing's torsional vibrations about the affine space quarter-chord.  $M_{\alpha_0}$  is the running aerodynamic moment coefficient due to the wing's torsional vibrations about the affine space quarter-chord. These coefficients, which are functions of the affine space reduced frequency (Strouhal number),  $k_0$ , are expressed as follows:

$$\begin{aligned} L_{h_0} &= 1 - \frac{2i}{k_0} \bar{C}(k_0) \\ L_{\alpha_0} &= \frac{1}{2} - \frac{i}{k_0} [1 + 2\bar{C}(k_0)] - \frac{2\bar{C}(k_0)}{k_0^2} \\ M_{\alpha_0} &= \frac{3}{8} - \frac{i}{k_0} \end{aligned} \quad (10)$$

where:

$C(k_0)$  is the Theodorsen's function in the affine space given by:

$$\bar{C}(k_0) = \frac{H_1^2(k_0)}{[H_1^2(k_0) + iH_0^2(k_0)]} \quad (11)$$

$H_0^2$  and  $H_1^2$  are Hankel functions and  $k_0$  is given by:

$$k_0 = \frac{\omega c'_0}{U_0} \quad (12)$$

$U_0$ , the affine space flight velocity, is given by:

$$U_0 = rU \quad (13)$$

The Hankel functions in Equation (11) are usually available in tabulated forms. Hence  $L_{h_0}$ ,  $L_{\alpha_0}$ , and  $M_{\alpha_0}$ , can be calculated for a chosen value of  $k_0$  from which  $L_0$  and  $M_0$  could be evaluated for flutter analysis.

It must be pointed out that the affine space unsteady aerodynamics is not restricted to two-dimensional space (airfoil section), but could also be evaluated in three-dimensional space with various levels of sophistication (including nonlinear treatments).

### 5.3.5 Flutter Analysis

Without any loss of generality, Equation (6) shall be specialized here to deal with the flutter arising from the coalescence between the first bending and the first torsional frequencies of the wing. Hence Equations (6), no matter what

order of approximation was used for estimating the uncoupled natural frequencies, reduce to:

$$\begin{aligned} m_0 \ddot{h}_0 + S_{\alpha_0} \ddot{\alpha}_0 + m_0 \omega_{h1}^2 h &= L_0 \\ S_{\alpha_0} \ddot{h}_0 + I_{\alpha_0} \ddot{\alpha}_0 + I_{\alpha_0} \omega_{\alpha 1}^2 \alpha_0 &= M_0 \end{aligned} \quad (14)$$

where:

$$\begin{aligned} m_0 &= \rho_0 a_1, S_{\alpha_0} = \rho_0 a_2 = m_0 e x_{\alpha_0} \\ I_{\alpha_0} &= \rho_0 a_3 = m_0 e^2 r_{\alpha_0}^2 \end{aligned} \quad (15)$$

$r_{\alpha_0}$  is basically a non-dimensionalized radius of gyration in the affine space.

Using the simple harmonic motion assumptions in Equation (9), non-dimensionalized closed form flutter equations resulting from Equations (8) and (13) are:

$$\begin{aligned} &\left\{ \frac{r_{m_0}}{\pi \bar{\rho} c_0'^2} \left[ 1 - (f \bar{\omega})^2 \right] + L_{h_0} \right\} H_0 + \\ &\left\{ \left[ L_{\alpha_0} - L_{h_0} \left( e + \frac{1}{2} \right) \right] + \frac{x_{\alpha_0} r_{m_0}}{\pi \bar{\rho} c_0'^2} \right\} \bar{\alpha}_0 = 0 \\ &\left\{ \left[ \frac{1}{2} - L_{h_0} \left( e + \frac{1}{2} \right) \right] + \frac{x_{\alpha_0} r_{m_0}}{\pi \bar{\rho} c_0'^2} \right\} H_0 + \left\{ L_{h_0} \left( e + \frac{1}{2} \right)^2 + \right. \\ &\left. M_{\alpha_0} - \left( L_{\alpha_0} + \frac{1}{2} \right) \left( e + \frac{1}{2} \right) + \frac{r_{\alpha_0}^2 r_{m_0}}{\pi \bar{\rho} c_0'^2} \left[ 1 - \bar{\omega}^2 \right] \right\} \bar{\alpha}_0 = 0 \end{aligned} \quad (16)$$

where:

$$\bar{\omega} = \frac{\omega_{\alpha_0 1}}{\omega} \quad (17)$$

$$f = \frac{(.597)^2 \pi}{\frac{L_0}{c_0} \sqrt{24 D_0^*}} \quad (18)$$

or

$$f = \frac{(.597 \pi)^2}{\frac{L_0}{c_0} k_{\alpha_0 1}} \quad (19)$$

where:

Equations (18) and (19) give the St. Venant's approximation and the present approximation respectively. Clearly Equation (16), which can be put in the form,

$$\begin{bmatrix} M \end{bmatrix} \begin{bmatrix} H_0 \\ \bar{\alpha}_0 \end{bmatrix} = 0 \quad (20)$$

are eigenvector equations. Hence for nontrivial solutions the determinant of  $M$  must vanish identically. The fact that  $M$  is complex (resulting from the non-dimensionalized unsteady aerodynamic functions) makes the computations of the eigenvalues for a set of configuration parameters a tedious exercise.

The computations were carried out by selecting sets of configuration parameters and solving for  $k_0, r_{\alpha_0}, M_0, x_{\alpha_0}$  and  $\bar{\omega}$ . These parameters are the affine space quantities.

where:

$$\mu_o = \frac{r_{m_o}}{\pi \bar{\rho} c_o^2}$$

### 5.3.6 Results and Conclusions

The computations of this flutter analysis to study the effects of proper representation of the wing's constrained warping phenomenon is still in progress. However the results obtained thus far which appear to be interesting, permit the following conclusions or observations on the mechanism of flutter for flat plate models of aircraft wings fabricated of composite with a more realistic (or accurate) representation of the constrained warping phenomenon.

If  $U_f$  and  $U_{f_{STV}}$  are the flutter speeds predicted by the present more sophisticated theory and St. Venant theory (which neglects warping constraint) respectively, and  $r_{\alpha_o}, \mu_o$  and  $x_{\alpha_o}$  are the radius of gyration, mass and static mass unbalance ratios respectively, Figures 2 thru 6 can be used to draw the following conclusions for the range of parameters investigated: (i) significant errors (conservative and nonconservative) in flutter speeds are possible if warping is neglected in composite wing flutter analysis, (ii) the errors decrease with the generic design variable  $Do^*$  (unless there is a low static mass unbalance) and an effective aspect ratio  $\ell_o/c_o$ , (iii) flutter speed errors due to neglect of warping constraint tend to be greater for a system with less static mass unbalance, (iv) while conservative flutter speed errors ( $U_f/U_{f_{STV}} < 1$ ) appear to be the trend in systems with high static mass unbalance, nonconservative flutter speed errors ( $U_f/U_{f_{STV}} > 1$ ) are possible in systems with low static mass unbalance.

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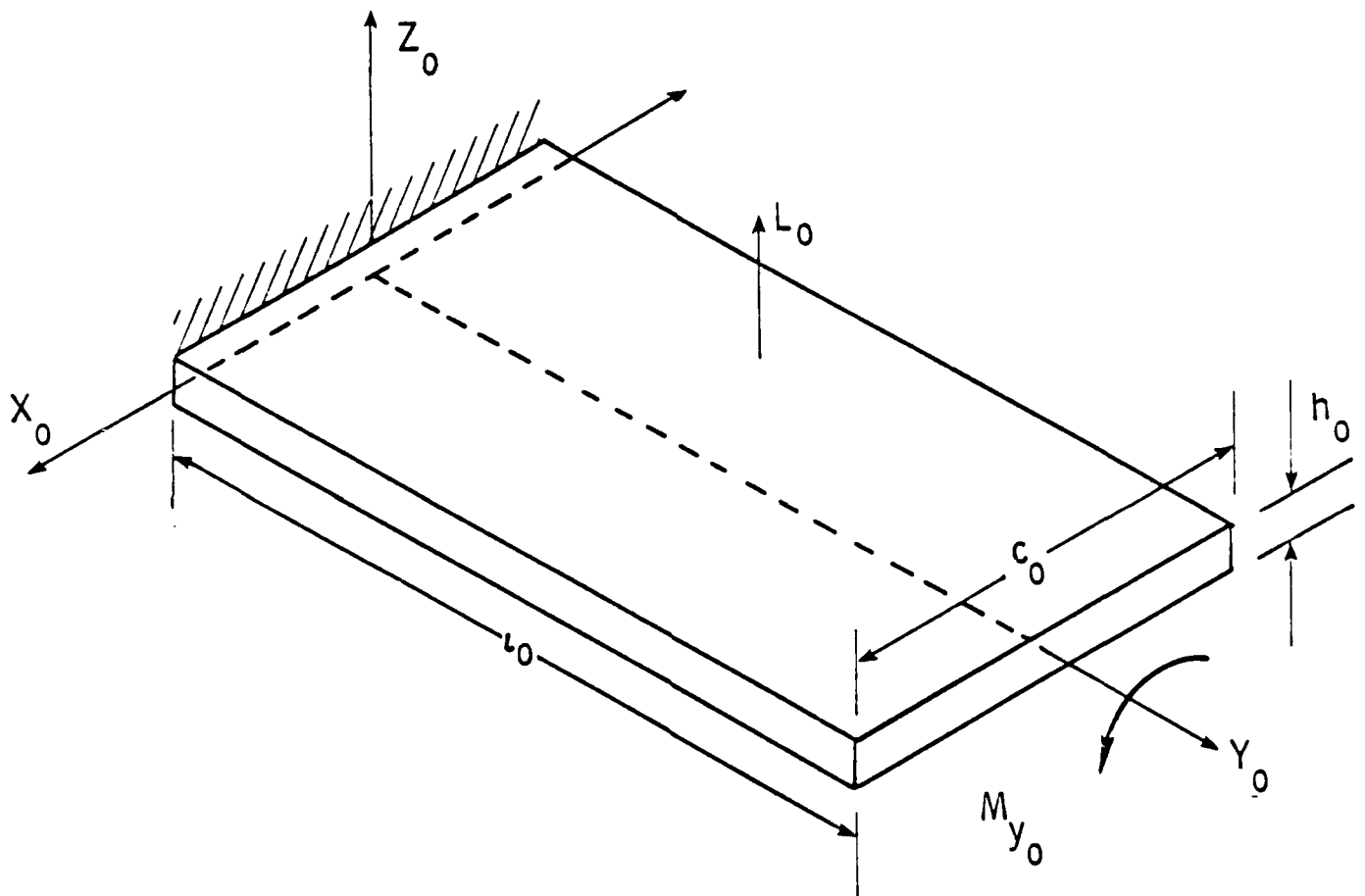


Figure 1

# RATIO OF FLUTTER SPEED WITH WARPING CONSTRAINT TO FLUTTER SPEED WITHOUT WARPING CONSTRAINT

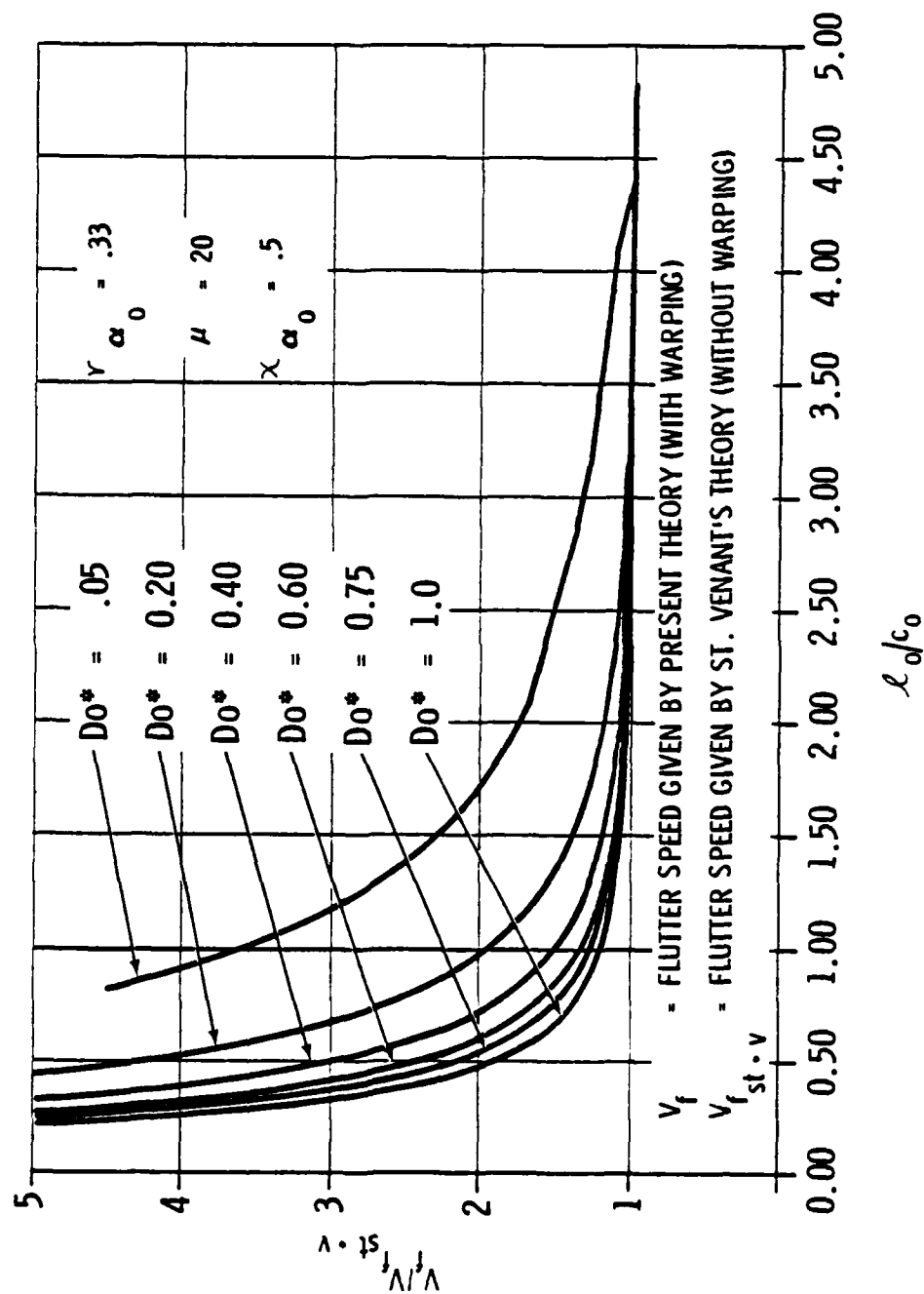


Figure 2

# RATIO OF FLUTTER SPEED WITH WARPING CONSTRAINT TO FLUTTER SPEED WITHOUT WARPING CONSTRAINT

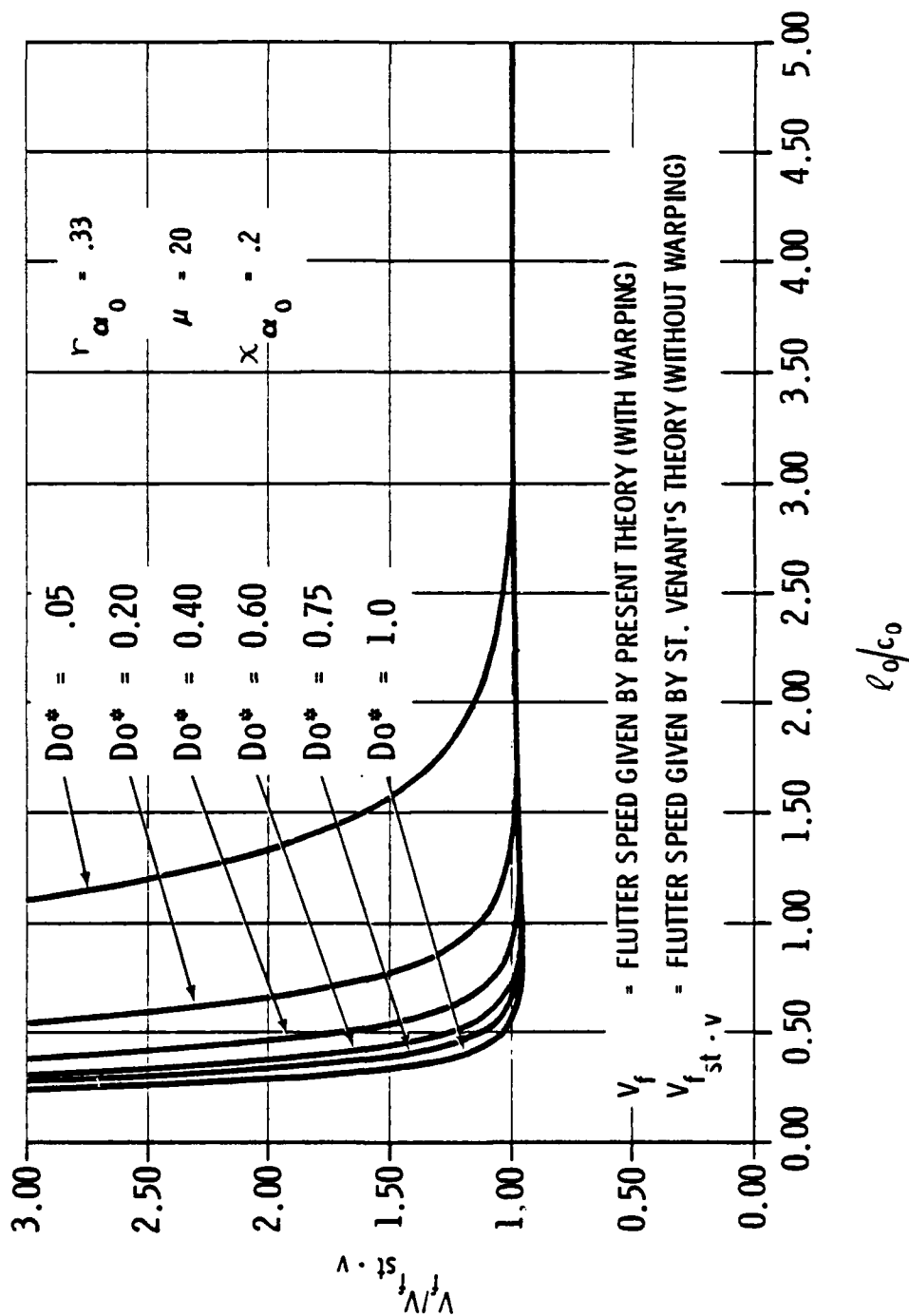


Figure 3

# RATIO OF FLUTTER SPEED WITH WARPING CONSTRAINT TO FLUTTER SPEED WITHOUT WARPING CONSTRAINT

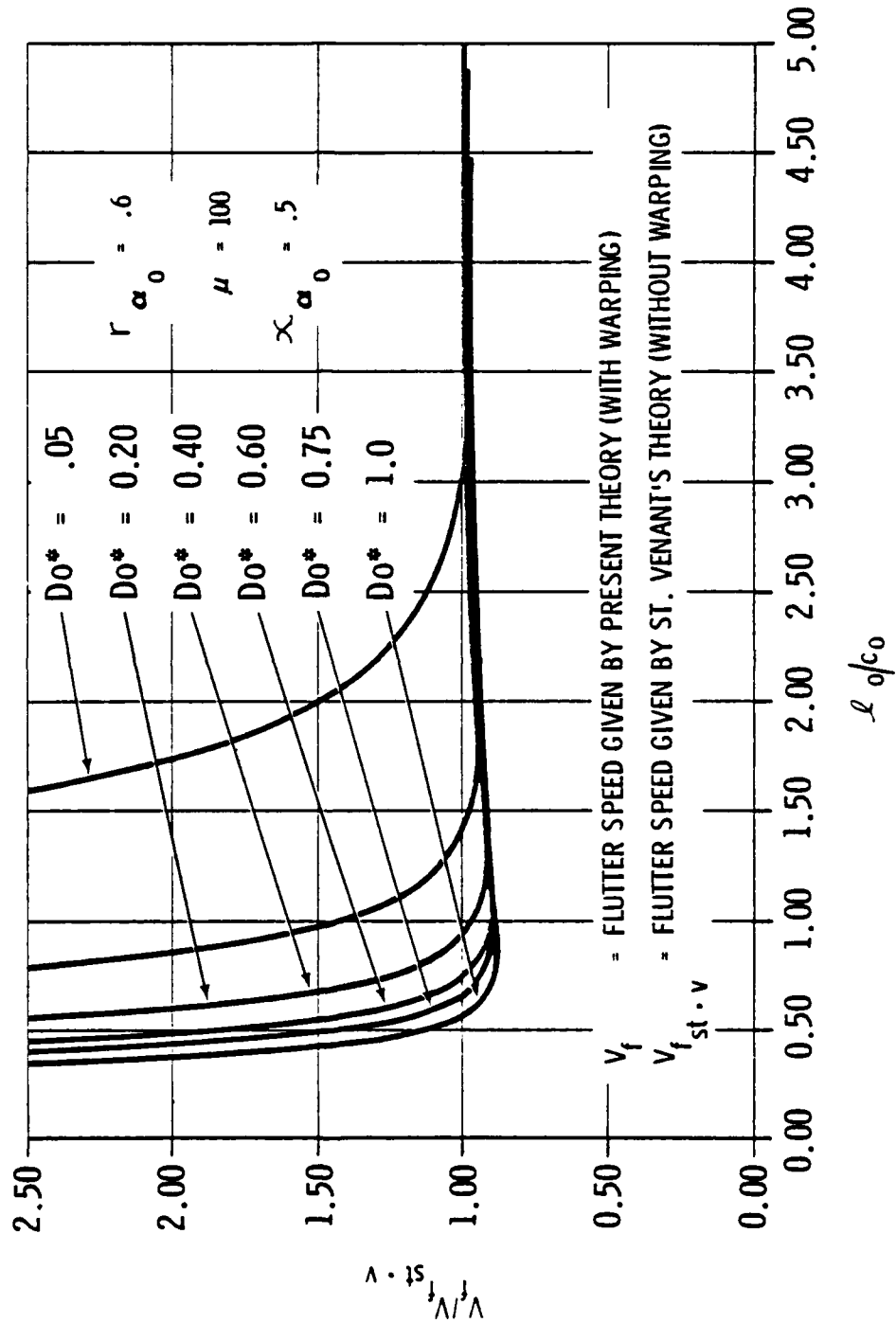


Figure 4

# RATIO OF FLUTTER SPEED WITH WARPING CONSTRAINT TO FLUTTER SPEED WITHOUT WARPING CONSTRAINT

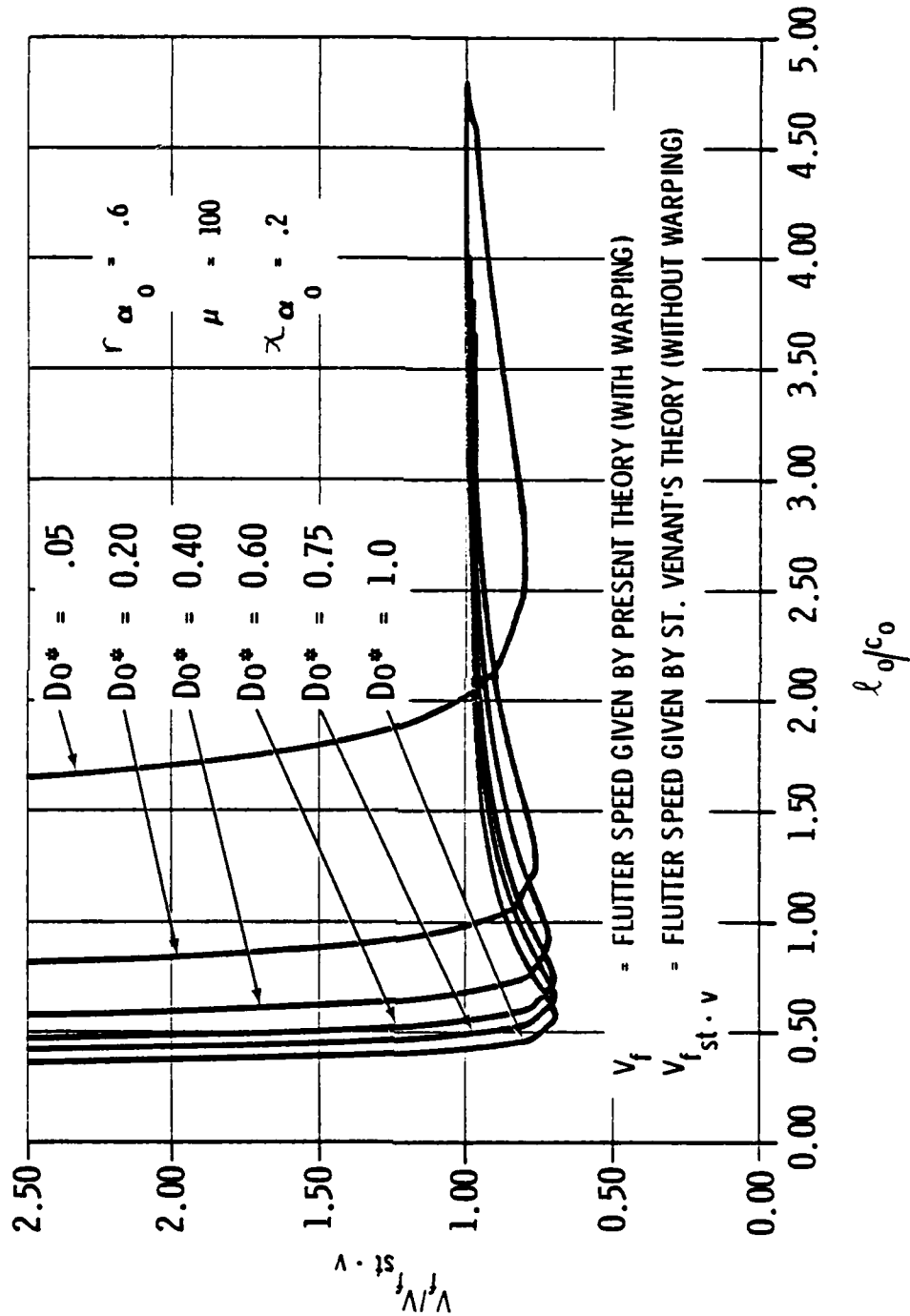


Figure 5

# RATIO OF FLUTTER SPEED WITH WARPING CONSTRAINT TO FLUTTER SPEED WITHOUT WARPING CONSTRAINT

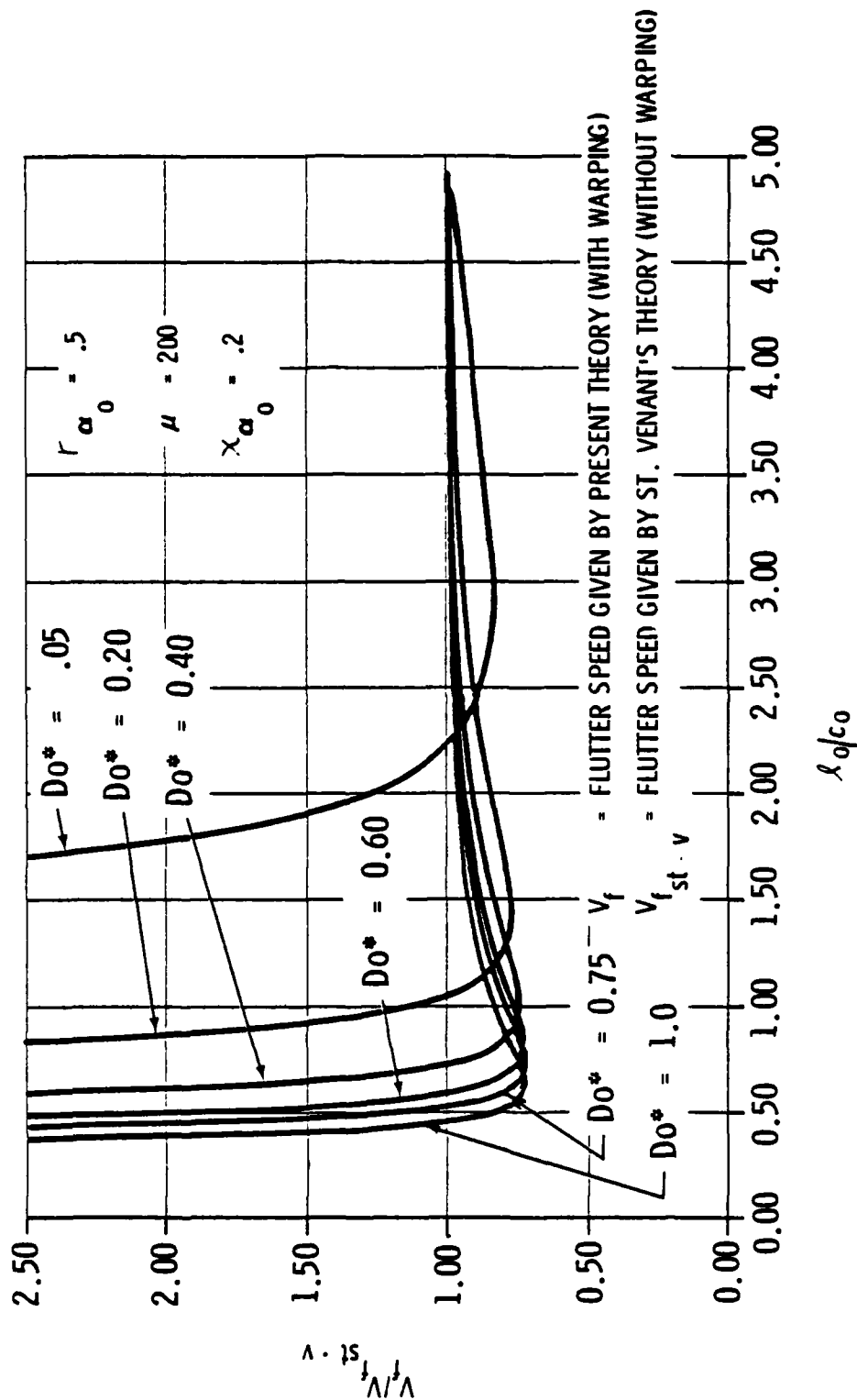


Figure 6

#### 5.4 CLOSED FORM DYNAMIC SOLUTIONS FOR SUPERMANEUVERABLE AIRCRAFT WINGS by Gabriel A. Oyibo

##### 5.4.1 Introduction

The performance of modern supermaneuverable aircraft can be made to benefit a great deal from significant advances in materials technology and the availability of more accurate aerodynamic prediction capabilities. Supermaneuverability, as a design goal, invariably calls for an optimization of the design parameters. Optimization may be partially accomplished, for example, by using composite materials to minimize weight. Indeed, it is known that composite materials can be tailored to resolve the dynamic or static instability problems of these types of aircraft. This concept is referred to as aeroelastic tailoring.

While aeroelastic tailoring can provide tremendous advantages in the design of an aircraft, the analysis which provides the basis for the aeroelastic tailoring itself is generally very complex. This is rather unfortunate, since a good fundamental physical insight of the tailoring mechanism is required for accurate and reliable results and earlier theories do not provide sufficient insight.

Although earlier studies(1,2,3) have indicated that the St. Venant's torsion theory is reasonably accurate except for aircraft wings with fairly low aspect ratios, the theory supporting that conclusion was based on the assumption that the wing is constructed of isotropic materials. Basically, the St. Venant's torsion theory assumes that the rate of change of the wing's twist angle with respect to the spanwise axis is constant. This assumption is hardly accurate particularly for modern aircraft construction in which different construction materials are employed and the aerodynamic loads vary significantly along the

wing's span. However, References 1-3 have shown that (in spite of such an inaccurate assumption) the main parameter that determines the accuracy of the St. Venant's theory is the wing's aspect ratio. Thus, it was determined that the theory is fairly accurate for moderate to high aspect ratio wings constructed of isotropic materials. In recent studies(4,5,6) however, it has been shown that for wings constructed of orthotropic composite materials, the conclusion of References 1-3 needs to be modified. Rather than using the geometric aspect ratio of the wing to determine the accuracy of St. Venant's twist theory, it was suggested that a generic stiffness ratio as well as an effective aspect ratio which considers the wing's geometry and the ratio of the principal directional stiffness should be considered in establishing the accuracy of St. Venant's theory.

In Reference 7, an attempt was made to look at some dynamic theories that can be used to understand the aeroelastic tailoring mechanism. Specifically, the accuracy of the St. Venant torsion theory which is relatively simple and frequently used in aeroelastic analysis is examined with particular reference to the effects of the wing's aspect ratio, as well as other physical parameters. In the study (7) the first task was to examine the role of coupling (both mass and elastic coupling) on the accuracy of St. Venant's theory applied to static problems. It was discovered that coupling plays a very significant role on the accuracy of St. Venant's twist theory. The second task was to investigate the torsional vibration for a flat plate model of an aircraft wing fabricated of composite materials in which the warping phenomenon is more realistically represented, with particular emphasis on higher frequencies, and to compare results with those from a representation based on St. Venant's theory. Similarly, Reference 8 examined the effects of the constraint of warping on the flutter for these models of composite wings.

During the discussion of the first quarterly Technical Report (7,8), which took place on 5 September 1985 at the office of Dr. Anthony K. Amos (the program monitor and a Program Manager for AFOSR), questions that were raised included one that dealt with the following observation: in studying the effects of the warping constraint on the dynamics and aeroelastic stability of plate-like models of composite aircraft wings, using the new methodology developed at Fairchild Republic Company (FRC), along with wing displacement assumptions in which chordwise curvatures were ignored, it appeared startling to notice from the results obtained that the dynamic and aeroelastic characteristics of these wings are dependent on their chordwise stiffnesses.

During the month of September, a more detailed explanation of this apparent contradiction was formulated and the following conclusions were arrived at:

- a) The dynamic and aeroelastic characteristics of these aircraft wings are only implicitly (not explicitly) dependent on the chordwise stiffnesses.
- b) This implicit relationship is brought about by Poisson effects.
- c) The new methodology appears to expose certain details that may not be visible when conventional methods are employed to analyze composite aircraft dynamic and aeroelastic phenomena.

The difficult task of determining closed-form solutions to the dynamic problem of composite aircraft wings with the coupling and warping constraints was also begun this quarter. Encouraging progress has been achieved on this task, which apparently has never been previously accomplished. The accomplishment of the task will provide the aerospace scientific community with new physical insights into these dynamic and aeroelastic phenomena. It should also provide useful background for properly investigating the effects of chordwise curvatures on these phenomena, which is planned to be the next task.

#### 5.4.2 Formulation

Consider an aircraft wing fabricated of composite materials and mathematically idealized as a cantilevered plate subjected to an aerodynamic flow over its surfaces. The mathematical statement of the virtual work theorem for such a plate model is well known and documented. It is also known that such mathematical statements of the virtual work theorem for a laminated plate model are characterized with the existence of so many variables (in the statement), reflecting the various directional properties for the laminated plate model, which would tend to interfere with any physical insight that might be desired from a phenomenological analysis employing such a mathematical statement. The newly discovered affine transformation concept (5,6) was developed principally to resolve such a problem.

This new concept therefore can be used to evolve the mathematical statement of the virtual work theorem in an affine space given by the following equation.

$$\begin{aligned} \delta \bar{U}_0 = 0 = & 2 \int_0^t \iint_A \left\{ (w_{,x_0x_0})^2 + 2D^* \left[ (1 - \epsilon)(w_{,x_0y_0})^2 \right. \right. \\ & \left. \left. + \epsilon w_{,x_0x_0} w_{,y_0y_0} \right] + (w_{,y_0y_0})^2 + L_1 w_{,x_0x_0} w_{,x_0y_0} + L_2 w_{,y_0y_0} w_{,x_0y_0} \right\} dx_0 dy_0 dt \\ & - \frac{\delta}{2} \int_0^t \iint_A \rho_0 \dot{w}^2 dx_0 dy_0 dt + \int_0^t \iint_A \Delta p_0 \delta w dx_0 dy_0 dt \end{aligned} \quad (1)$$

where:

$$\begin{aligned} \bar{U}_0 &= \frac{\bar{U}}{D_{22}} \left( \frac{D_{22}}{D_{11}} \right)^{1/4} ; \quad D^* = \frac{D_{12} + 2D_{66}}{(D_{11}D_{22})^{1/2}} ; \quad \epsilon D^* = \frac{D_{12}}{(D_{11}D_{22})^{1/2}} \\ L_1 &= \frac{4D_{16}}{(D_{11})^{3/4}(D_{22})^{1/4}} ; \quad L_2 = \frac{4D_{25}}{(D_{11})^{1/4}(D_{22})^{3/4}} \\ \Delta P_0 &= \frac{\Delta p}{D_{22}} ; \quad \rho_0 = \frac{\rho h}{D_{22}} \end{aligned} \quad (2)$$

$D_{ij}$  are the elastic constants,  $\rho$ , is the material density,  $\Delta p$  is the differential pressure distribution,  $w$  is the displacement,  $t$  is the time,  $A$  integrals represent area integrals and  $h$  is the wing box depth.

Equations (1) and (2) therefore form the basis of the newly developed methodology. The equations of motion of a plate model of an aircraft wing can now be derived by prescribing a realistic wing displacement and using Equation (1).

When Equation (1) is compared to its physical space counterpart, it is seen that Equation (1) has fewer variables. It is also seen that Equation (1) contains only non-dimensionalized stiffness quantities (compared to dimensional stiffness quantities in its physical space counterpart). Another feature of this new methodology which makes it unique is that the non-dimensionalization (a consequence of the affine transformation) is accomplished before assuming the wing deformations. This means that the non-dimensionalization is independent of how the wing deforms. A non-dimensionalization scheme that depends on a particular assumption of the wings deformations could lead the analyst to an

incorrect physical interpretation of results, since the wing's deformations assumptions have inherent errors because they are based on the analyst's judgments and experience. This observation may become clearer during the evolution of a warping parameter with which to study the effects of the warping constraint phenomenon on the status and dynamics of a wing fabricated of composite materials later in this study.

If the chordwise curvature is neglected in an initial approximation, the wing's deformation may be assumed as follows:

$$w(t, x_0, y_0) = h_0(t, y_0) + x_0 \alpha_0(t, y_0) \quad (3)$$

where  $h_0$  and  $\alpha_0$  are the bending and twisting displacements respectively.

It can be shown that when Equation (3) is substituted into Equation (1) and the variational calculus is carried out for arbitrary  $h_0$  and  $\alpha_0$ , the following equations of motion are obtained:

$$\begin{aligned} a_1 h_0^{iv} + a_2 \alpha_0^{iv} + a_5 \alpha_0^{iii} + \rho_0 a_1 \ddot{h}_0 + \rho_0 a_2 \ddot{\alpha}_0 &= L_0 \\ a_2 h_0^{iv} - a_5 h_0^{iii} + a_3 \alpha_0^{iv} - a_4 \alpha_0'' + \rho_0 a_3 \ddot{\alpha}_0 + \rho_0 a_2 \ddot{h}_0 &= M_0 \end{aligned} \quad (4)$$

where:

$$a_1 = \int \frac{\bar{c}_0}{e\bar{c}_0} dx_0 \quad ; \quad a_2 = \int \frac{\bar{c}_0}{e\bar{c}_0} x_0 dx_0$$

$$a_3 = \int \frac{\bar{c}_0}{e\bar{c}_0} x_0^2 dx_0 \quad ; \quad a_4 = 2 \int \frac{\bar{c}_0}{e\bar{c}_0} D^*(1-\epsilon) dx_0 \quad (5)$$

$$L_0 = \int \frac{\bar{c}_0}{e\bar{c}_0} \Delta p_0 dx_0 \quad ; \quad a_5 = \int \frac{\bar{c}_0}{e\bar{c}_0} L_2 dx_0$$

$$M_0 = \int \frac{\bar{c}_0}{e\bar{c}_0} x_0 \Delta p_0 dx_0$$

$$-\infty < e < 0 \quad ; \quad \bar{c}_0 = \frac{c_0}{1-e} \quad (6)$$

$$(\quad)' = \frac{\partial}{\partial y_0}, (\quad)^{\circ} = \frac{\partial}{\partial t}$$

### 5.4.3 Evolution of Warping Parameters

The evolution of the warping parameter with which to study the aeroelastic warping constraint phenomenon for wings fabricated of composite materials is a process that depends on the sophistication of the wing's mathematical model; whether coupling effects are included, whether the wing's chordwise curvatures are included and so on. Therefore, any warping parameter is as good as the corresponding wing's displacements assumptions. However, Equation (1) makes it possible for the analyst to determine its effective independent variables even before the displacement assumptions are made.

By non-dimensionalizing the spanwise space variable in Equation (4), depending on whether one is interested in the static, dynamic, coupled or uncoupled displacements, one of the following warping parameters may be useful.

$$\lambda_c = \frac{\ell_0}{c_0} \sqrt{\frac{3}{2} D_0^*} \quad (7)$$

$$\bar{\lambda}_c = \frac{\ell_0}{c_0} \sqrt{\frac{3}{2} (D_0^* - \frac{L^2}{8})} \quad (8)$$

$$\bar{\lambda}_c = \frac{\ell_0}{c_0} \sqrt{\frac{3}{2} \frac{D_0^*}{1 - 3\bar{a}_2^2}} \quad (9)$$

where

$$D_0^* = D^* (1 - \epsilon) ; \quad \bar{a}_2 = \frac{a_2}{c_0} \quad (10)$$

$(\ell_0/c_0)$  is defined as the wing's effective aspect ratio and  $D_0^*$  and  $L$  are the generalized stiffness and coupling ratios respectively (defined in earlier work such as References 5 and 6).

Equations (7) thru (9) represent the appropriate warping parameter for dynamic deformation, static displacement with elastic cross-coupling, and static deformation with "geometric" coupling ( $\epsilon \neq 1$ ) respectively.

It was discovered in References 7 and 8 that evolving the warping parameter in a manner shown in Equations (1) thru (3), should enable one to effectively investigate the effects of warping on the composite wing's dynamics (or the accuracy of St. Venant's theory. From the lamination theory for composites it is known that while  $D_0^*$  and  $(\ell_0/c_0)$  are always positive,  $L$  and  $\bar{a}_2$  can be positive or negative. However, from Equations (1) and (3), it is clear that whether a composite wing has positive or negative coupling, the warping effect (in terms of  $\lambda_c$ ) is unchanged.

#### 5.4.4 Results and Conclusions

By using the evolved warping parameters defined in Equations 7 and 8 and appropriate boundary conditions, the boundary value or the eigenvalue problems associated with Equation 4 are solved in a closed-form manner to determine the wing's coupled vibration frequencies. The presence of the elastic coupling was found to significantly complicate the mathematical analysis, required to solve the problem in a closed-form manner. This fact therefore, is probably the principal reason why no one seems to have been successful previously in obtaining closed-form solutions. The necessity for closed-form solutions stems from what appears to be a lack of identified coherent trends which tend to characterize the results obtained from most of the numerical methods. That is, the earlier numerical solutions seem to be a collection of individual isolated results with no discernible trends exhibited which would unify the meaning of the results. In the process of trying to accomplish this task, the following phenomena were discovered: (a) the coupling mechanism transforms the bending-torsion oscillations (which are separately conservative problems) into a non-conservative problem, resulting in a complex determinant for the eigenvalues; (b) one "search" variable (rather than two) can be used to determine the eigenvalues (or zeros)

of this complex determinant, and (c) the frequency parameters are only functions of an effective aspect ratio ( $\ell_0/c_0$ ), a nondimensionalized coupling parameter,  $L_2$ , and a non-dimensionalized twisting stiffness parameter,  $Do^*$ , which is proportional to the warping parameter derived earlier.

Figure 1 shows a typical behavior of the real and imaginary parts of the determinant, which coalesce at its eigenvalues (zeros).

These figures therefore depict the mechanism by which coupling ( $L_2$ ) and the warping constraint (proportional to  $Do^*$ ) transform the first vibrational mode. The word transform has been utilized in order to make the point that the physical mechanism known as coupling transforms the nature of the first vibrational mode in the process of transferring energy to other modes. The following observations can be obtained from Figures 2 - 7: the first coupled frequency parameter (i) increases with decreasing coupling ( $L_2$ ), (ii) increases with increasing  $Do^*$ , (iii) is independent of  $Do^*$  for very small  $L_2$ , (iv) increases more rapidly with  $Do^*$  for larger coupling (v) decreases more rapidly with increasing  $L_2$  for lower  $Do^*$ . Observation (i) is supported by data computed numerically at MIT<sup>4</sup>, observation (ii) implies that the first coupled frequency which is supposed to be dominated by the first bending frequency, becomes more "torsional" as coupling is increased, observation (iii) indicates that as  $L_2$  approaches zero, the first coupled frequency approaches the pure (uncoupled) bending frequency, hence it becomes independent of  $Do^*$ ; this might provide the basis to explain the modal "transformations" reported for instance, by the study at MIT<sup>9</sup>. Figures 2 - 7 therefore seem to suggest a potentially accurate method to trace the origin of a coupled mode.

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$$\begin{aligned} L_2 &= 0.2 \\ D_0^* &= 0.7 \\ \ell_0/c_0 &= 0.3 \end{aligned}$$

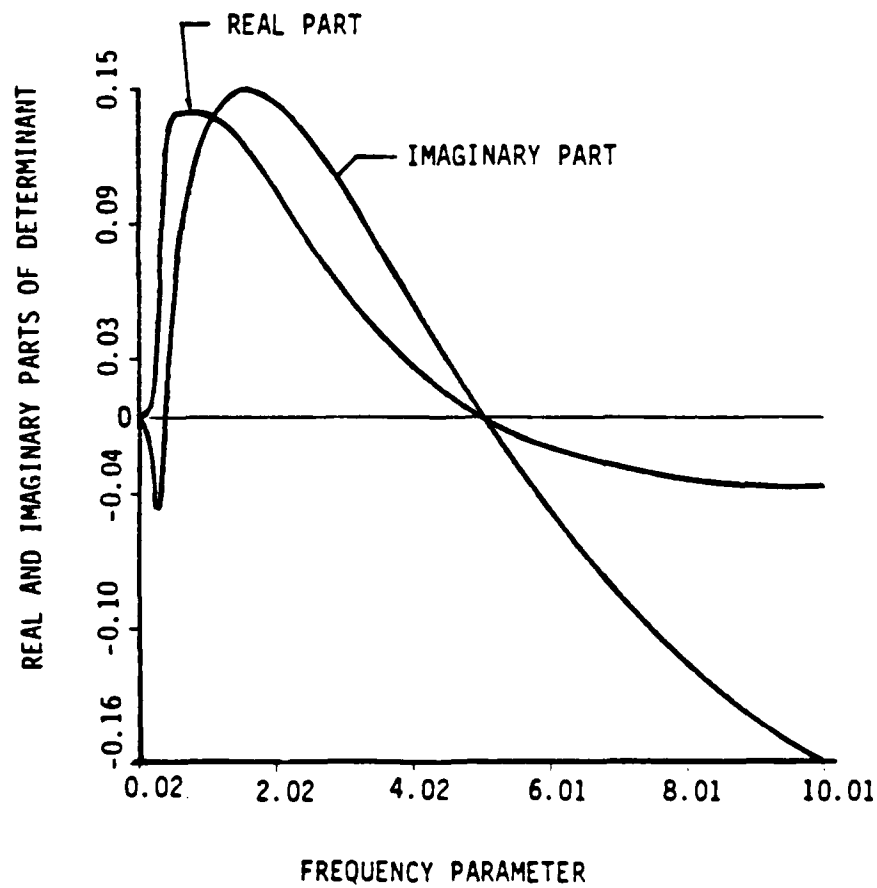


Figure 1. Behavior of the Complex Determinant for the Eigenvalues

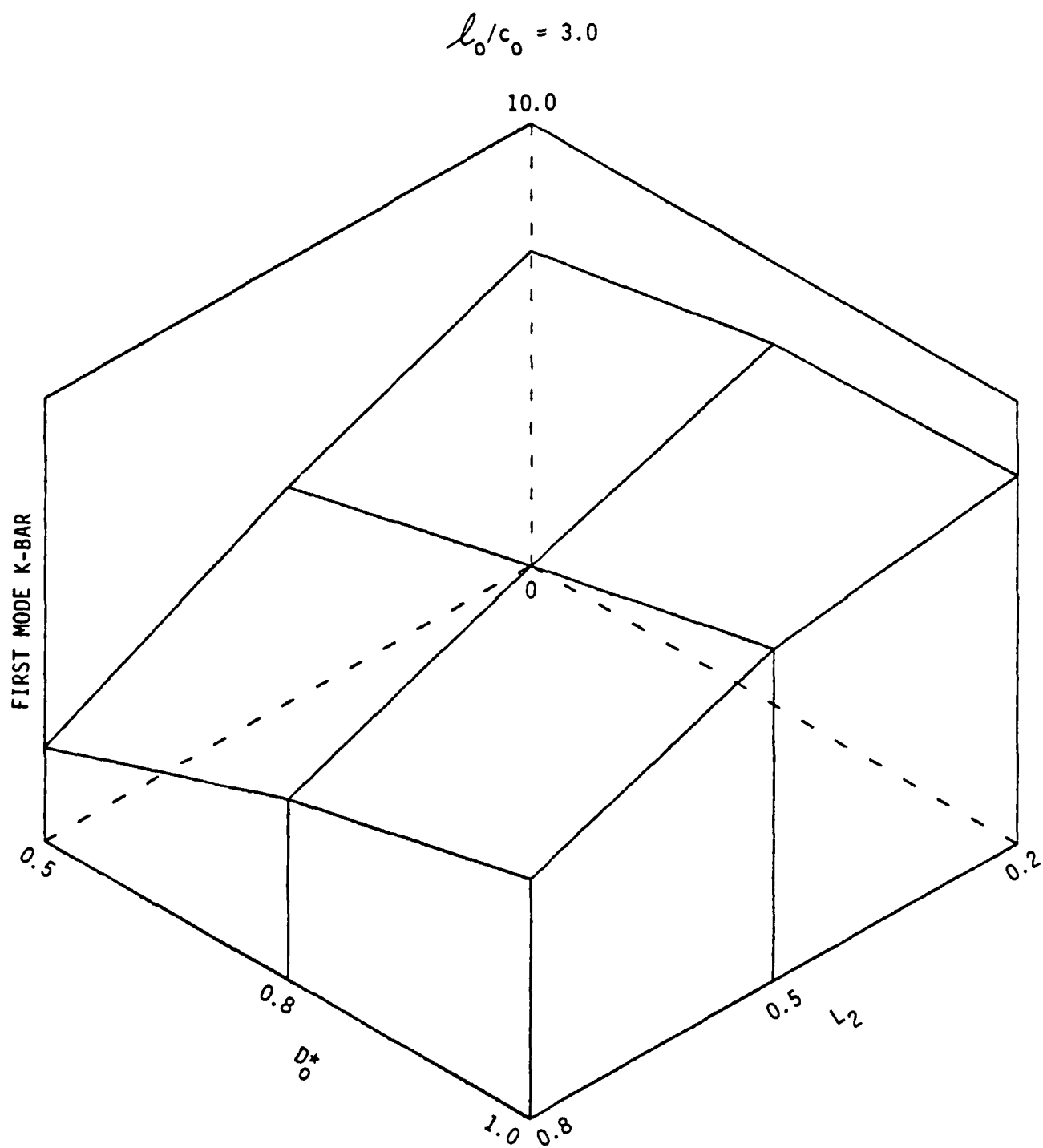


Figure 2. Behavior of First Coupled Frequency for  $\ell_0/c_0 = 3.0$

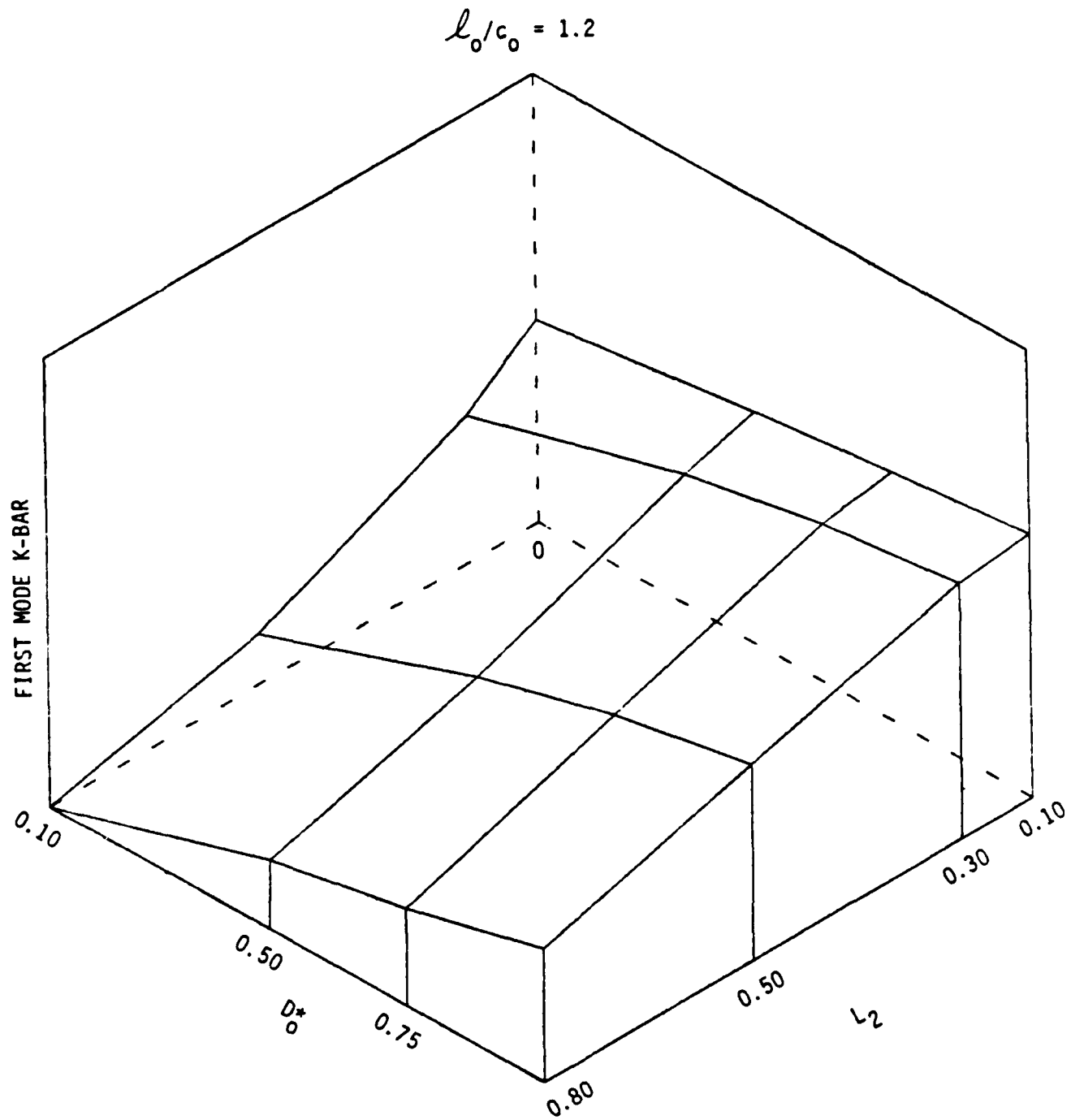


Figure 3. Behavior of First Coupled Frequency for  $\ell_0/c_0 = 1.2$

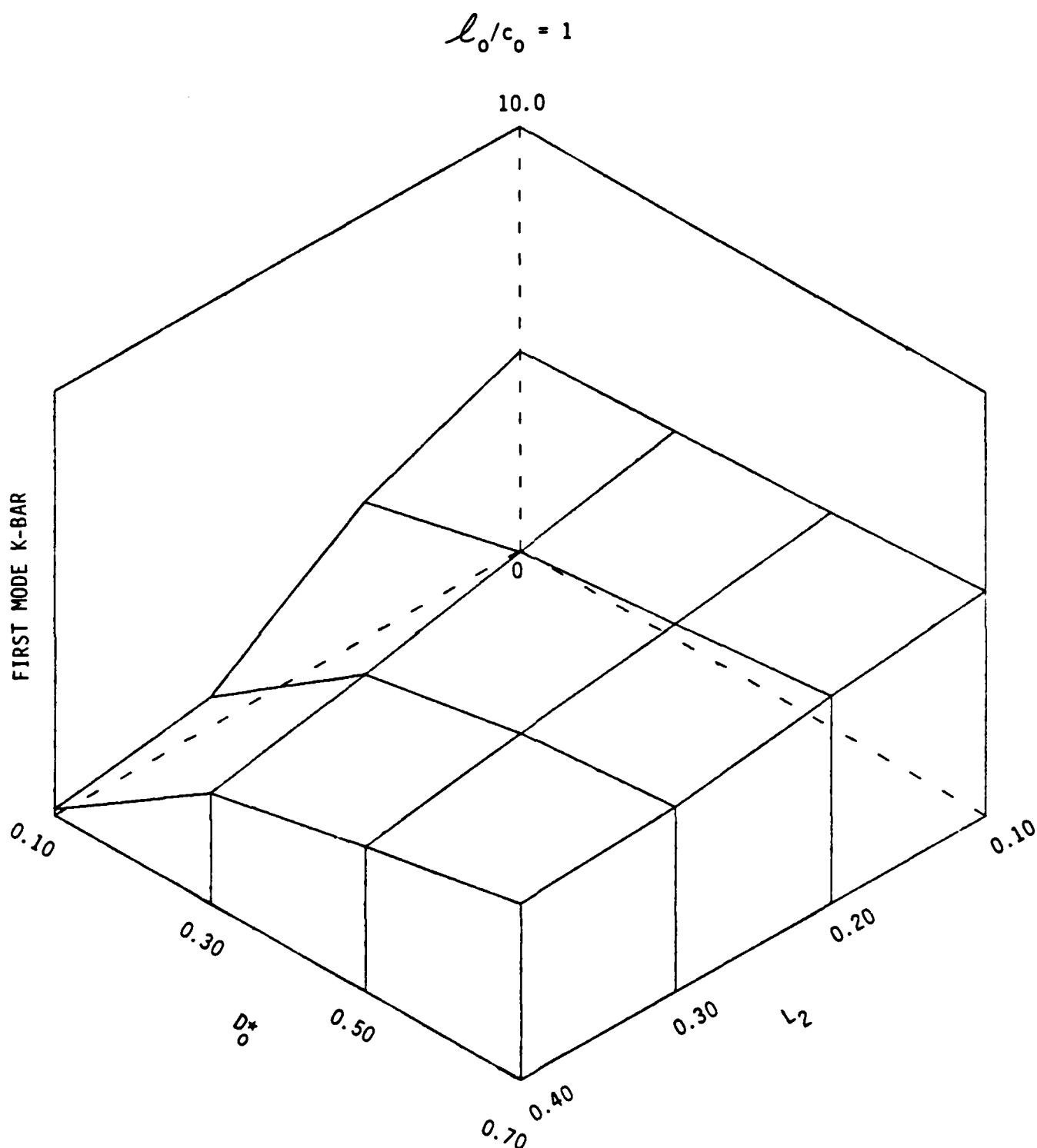


Figure 4. Behavior of First Coupled Frequency for  $\ell_0/c_0 = 1$

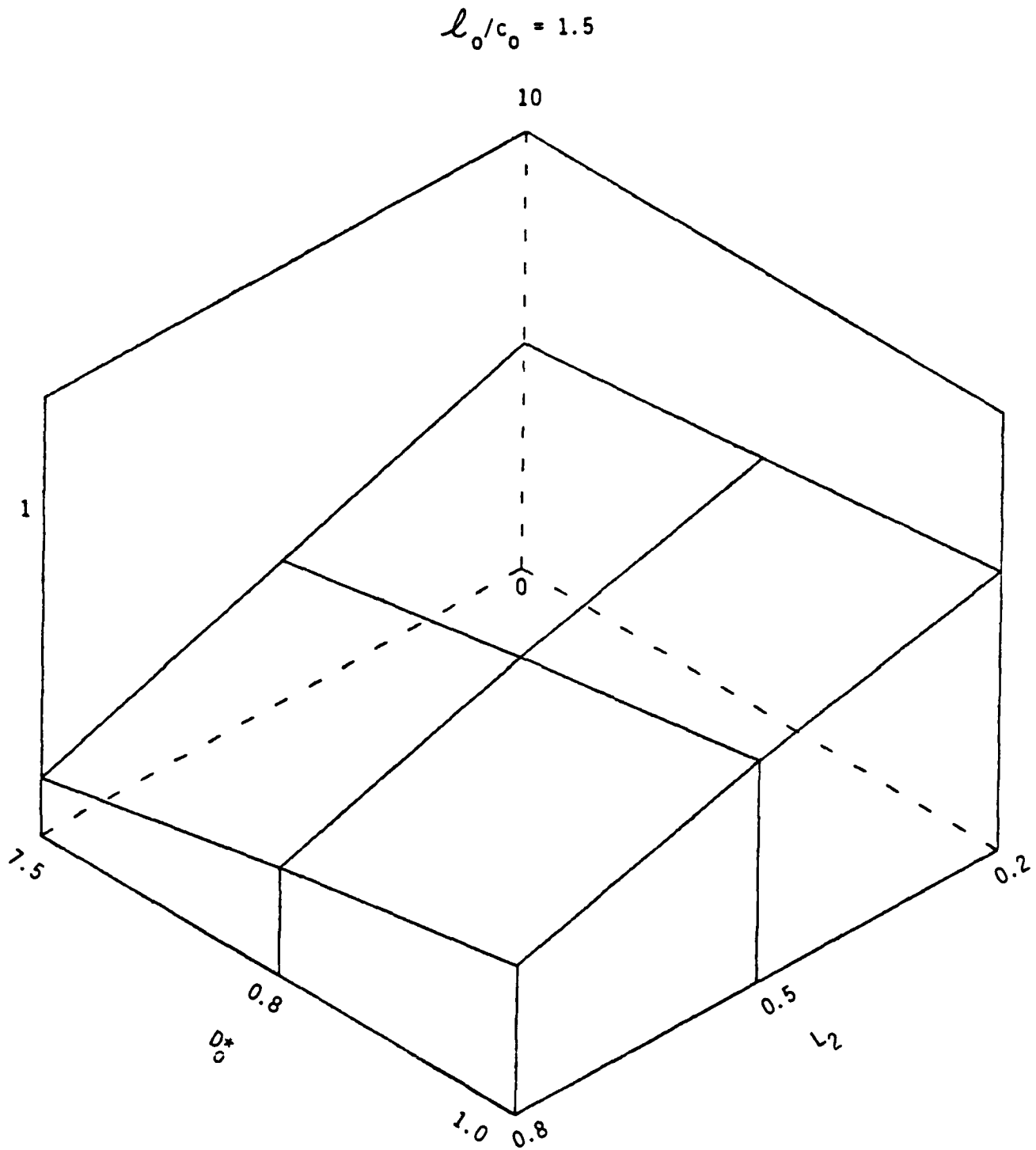


Figure 5. Behavior of First Coupled Frequency for  $\ell_0/c_0 = 1.5$

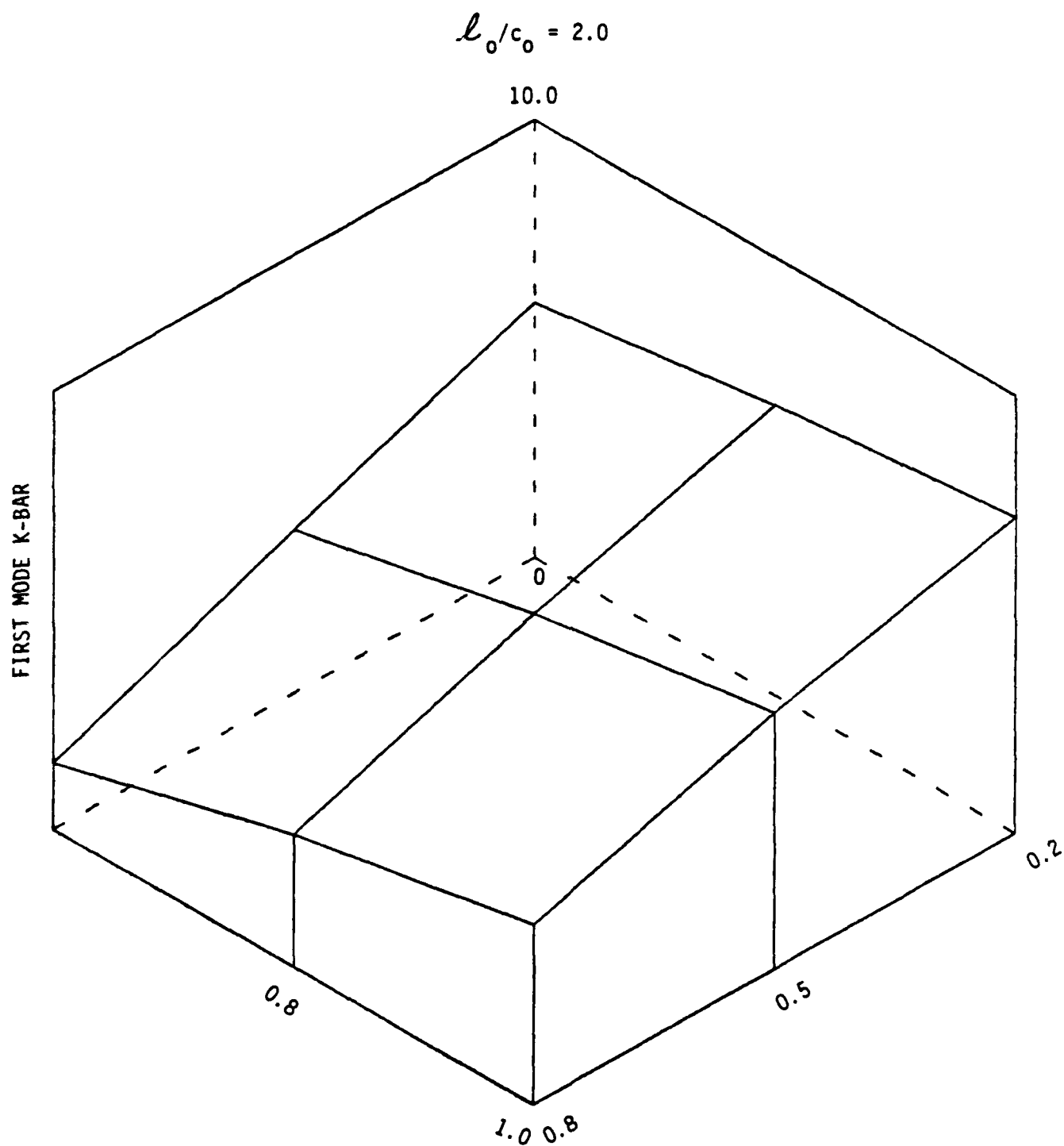


Figure 6. Behavior of First Coupled Frequency for  $\ell_0/c_0 = 2.0$

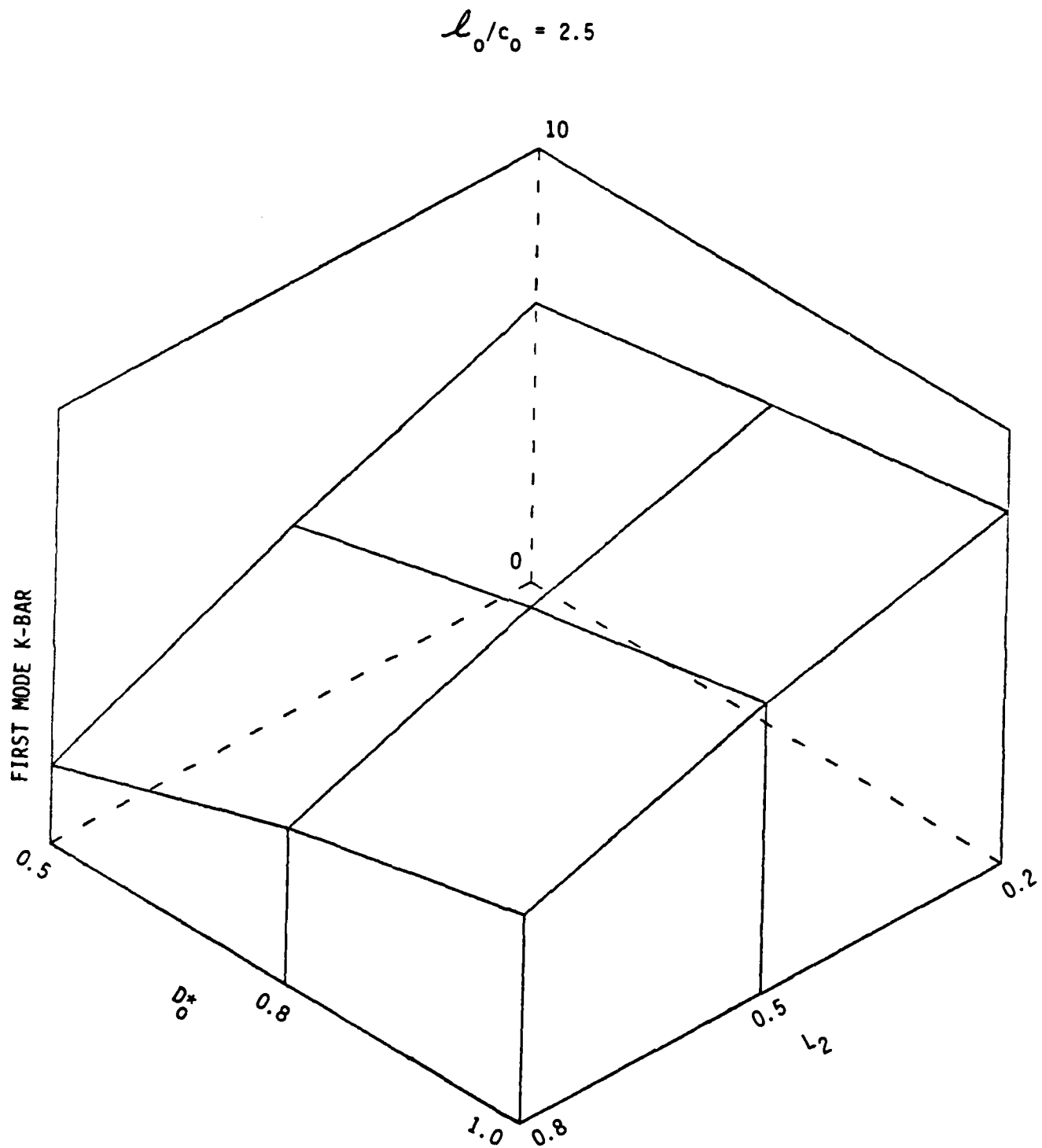


Figure 7. Behavior of First Coupled Frequency for  $l_0/c_0 = 2.5$

## 5.5 AEROELASTIC DIVERGENCE OF ORTHOTROPIC PLATES by Terrence A. Weisshaar

### 5.5.1 Introduction

The aeroelastic stability of lifting surfaces constructed of laminated composite materials has been the subject of numerous research studies during the past decade. A variety of mathematical models have been used for these studies; the choice of model has been dependent upon the objectives of the researcher. For instance, for preliminary design or trend studies the level of complexity is usually kept to a minimum, consistent with the experience of the analyst. Aeroelasticians have only recently begun to realize that some of their lengthy experience or "rules of thumb" accumulated with metallic, isotropic structures may not be strictly applicable to advanced composite materials. In particular, shearing strains in laminated materials, while safely neglected for certain loadings of metallics, assume great importance in composites. For instance, the ratio  $E/G$  is, for a homogeneous isotropic material, fixed to be  $2(1 + \mu) = 2.6$ ; a similar ratio for a laminated material may be of the order of 20 to 50. As such, effects of shearing are likely to propagate quite far from their source. This "source" is usually a restrained boundary or a concentrated load.

To illustrate the potential importance of shear flexibility, this study will examine the idealized problem of static aeroelastic divergence of an unswept, constant property, rectangular flat plate constructed of laminated composite material. The laminate is assumed to be orthotropic with respect to the x-y axis system shown in Figure 1.

The stiffness characteristics of the plate are given by the plate stiffness matrix  $[D_{ij}]$ . The out-of-plane deformation  $w(x,y)$  is positive downward,

along the +z axis formed by applying the right-hand rule to the x-y axes. A moving airstream with velocity V indicated in Figure 1 creates lift that acts on this plate.

The divergence of such a structure has been studied extensively. The most common academic problem models the plate as a beam-like structure using Engineers' beam bending and twist theories (Bredt-Batho theory). This theory neglects the inplane (x-y) normal stresses and restraint caused by the fixed edge of the plate and assumes that the cross-sections are free to warp out of the y-z plane. Neglect of this so-called "restraint of warping" effect does not lead to serious consequences for metallics if the planform aspect ratio is greater than about 6. This rule of thumb (aspect ratio greater than 6) appears to originate from an early study by Reissner and Stein [1], who investigated the effect of "restraint of warping" upon plates with a variety of loadings, none of them aeroelastic. Whether or not this rule applies to laminated structures is of concern to modern structures researchers.

Crawley and Dugundji [2] as well as Oyibo and Berman [3] shared this concern about the effect of material properties upon the restraint of warping effect in flat plates. They studied the effect of a warping parameter upon the torsional natural frequencies of a rectangular flat plate. They concluded that, in addition to planform aspect ratio, the plate stiffness ratio  $D_{11}/D_{66}$  must be considered. This ratio, for isotropic metallic materials, is

$$\frac{D_{11}}{D_{66}} = \frac{E}{G} = 2(1 + \mu) \quad (1)$$

However, for a laminated plate, it is

$$\frac{D_{11}}{D_{66}} = \frac{\sum_{i=1}^N Q_{11} \gamma_i}{\sum_{i=1}^N Q_{66} \gamma_i} \quad (2)$$

$$\gamma_i = \int_i^{i+1} z^2 dz \quad (3)$$

The integral in Eqn. 3 extends over the thickness of the  $i$ th ply comprising an element of the laminate. The quantities  $Q_{11}$  and  $Q_{66}$  are ply elastic properties measured along orthotropic directions. These properties vary with ply orientation with respect to the  $x$ - $y$  axis system in Figure 1. The ratio in Eqn. 2 may be of far different magnitude than that in Eqn. 1.

Reference 2 solved, in approximate form, the equations for free torsional vibration of a laminated, orthotropic, cantilever plate. Included in the analysis was the definition of a non-dimensional warping parameter,  $\beta$ . References 2 and 3 show that the inclusion of restraint of warping effects leads to significantly higher torsional frequencies as compared to those predicted using Bredt-Batho or St. Venant torsion theory.

### 5.5.2 Objective

To illustrate the potential importance of laminate geometry to even the simplest aeroelastic stability problem, an example was studied. Results of this study were then compared to results using an elementary classical solution. Potentially important features of excluding "restraint of warping" effects are illustrated.

The example involves the prediction of the divergence dynamic pressure of an unswept, rectangular, orthotropic laminated plate. An idealization using plate theory to model the structural deformation was used, together with aerodynamic strip theory to model the airloads resulting from deformation. A closed-form solution for the divergence dynamic pressure is found and then compared to that obtained using conventional St. Venant torsion theory.

### 5.5.3 Analysis

The variational calculus is used to generate the governing ordinary differential equations (in terms of  $x$ ) and associated boundary conditions. The following definitions are used:

$$K = 2cD_{16}$$

$$EI = cD_{11}$$

$$GJ = 4cD_{66}$$

$$S_0 = \frac{D_{11}c^3}{12}$$

The governing differential equations are:

$$EIw_0^{IV} + K\theta^{''''} + qca_0\theta = 0 \quad (4)$$

$$S_0\theta^{IV} - GJ\theta^{''} - Kw_0^{'''} - qcea_0\theta = 0 \quad (5)$$

The associated boundary conditions are:

$$\text{At } x = 0: \frac{dw_0}{dx} = 0; w_0 = 0; \theta = 0; \frac{d\theta}{dx} = 0 \quad (6)$$

The last boundary condition at  $x = 0$ ,  $\theta'(0) = 0$ , is the result of the requirement that  $\frac{\partial w}{\partial x}(0) = 0$  at all points along the clamped edge. This produces two BC's for a plate instead of the usual single beam clamping condition. Boundary conditions at  $x = L$  are:

$$\begin{aligned}
 EIw_0'' + K\theta' &= 0 \\
 EIw_0'''' + K\theta'' &= 0 \\
 S_0\theta'' &= 0 \\
 -S_0\theta'''' + GJ\theta' + Kw_0'' &= 0
 \end{aligned} \tag{7}$$

These equations are greatly simplified if the bend/twist stiffness cross-coupling parameter,  $K$ , is zero, implying that the plate is orthotropic with respect to the  $x$ - $y$  axis system. With  $K = 0$ , the divergence equations for  $\theta$  can be solved independently of those for  $w_0$ . The  $\theta$  equations are:

$$S_0\theta^{IV} - GJ\theta'' - qcea_0\theta = 0 \tag{8}$$

$$\theta(0) = \theta'(0) = 0 \tag{9}$$

$$\theta''(L) = S_0\theta''''(L) - GJ\theta'(L) = 0 \tag{10}$$

The parameter  $S_0$  is indicative of the retention of the restraint of warping effect in this one-dimensional analysis. If  $S_0$  is zero then Eqn. 8 reduces to the classical differential equation for torsional divergence using St. Venant torsion theory. In addition, if  $S_0$  is zero, then the BC's  $\theta'(0) = 0$  and  $\theta''(L) = 0$  are not present.

To solve the equation set, Eqns. 8 - 10, we first non-dimensionalize by defining a new variable,  $\eta$ , as follows:

$$\eta = \frac{x}{L} \tag{11}$$

In addition, an eigenvalue  $\lambda$  is defined as

$$\lambda^2 = qcea_0 L^2 / GJ \quad (12)$$

A nondimensional warping parameter  $\beta$  used in Reference 2 is defined as:

$$\beta = \frac{S_0}{GJL^2} \quad (13)$$

This non-dimensional parameter,  $\beta$ , is defined in terms of plate constants and geometry as

$$\beta = \frac{1}{48} \left( \frac{D_{11}}{D_{66}} \right) \left( \frac{c^2}{L^2} \right) \quad (14)$$

The governing differential equation is now written as:

$$\beta \theta^{IV} - \theta'' - \lambda^2 \theta = 0 \quad (15)$$

with boundary conditions:

$$\theta = \theta' = 0 \text{ at } \eta = 0 \quad (16)$$

and

$$\theta''' = \beta \theta^{IV} - \theta' = 0 \quad (17)$$

The derivatives in these latter three equations are with respect to  $\eta$ .

Using standard techniques, it can be shown that the characteristic equation satisfying Eqns (15 - 17) for divergence is

$$2 + \left( \frac{f}{g} - \frac{g}{f} \right) \sin(g) \sinh(f) + \left( \frac{1}{g^2} + \frac{1}{f^2} \right) \cos(g) \cosh(f) = 0 \quad (18)$$

Where  $f$ ,  $g$  and  $\beta$  are related as follows:

$$f^2 = g^2 + 1/\beta \quad (19)$$

$$\lambda^2 = g^2(1 + \beta g^2) \quad (20)$$

$$\lambda^2 = f^2(\beta f^2 - 1) \quad (21)$$

Solution for  $\lambda^2$  proceeds as follows. First, choose a value of  $\beta$ . Because of Eqn. 19, with  $\beta$  fixed, only  $g$  or  $f$  is now independent. Choose  $g$  as the independent parameter and iterate to solve Eqn. 18. This was accomplished using a short computer code. A plot of  $\lambda$  (divided by  $\lambda_0$ ) versus  $\beta$  is provided in Figure 2. The parameter  $\lambda_0^2$  is equal to  $\pi^2/4$  and corresponds to the eigenvalue obtained when  $\beta = 0$ , the classical torsional divergence case with St. Venant torsion theory.

The dynamic pressure at divergence is proportional to  $\lambda^2$ . Figure 2 indicates that the inclusion of the restraint of warping effect increases the divergence dynamic pressure. More importantly, failure to account for the restraint of warping effect most likely will lead to predicted divergence dynamic pressures that are lower than those to be found in experimental tests. While Figure 2 indicates the effect of  $\beta$  upon the prediction of divergence dynamic pressure, it does not provide adequate insight into the physical behavior of the system. To acquire a more complete understanding of this restraint of warping effect on plate divergence, let us consider an additional set of examples.

Crawley and Dugundji [2] also present experimental data related to free vibrations of flat orthotropic plates. Their specimens include 8 plies thick Hercules ASI/3501-6 graphite epoxy. Plate stiffness ( $D_{ij}$ ) data from Ref. 2 can be used to illustrate the importance of orthotropy to restraint of warping effects.

Table 1 shows  $D_{11}$ ,  $D_{66}$  data for 3 laminates. The first laminate is designed to be stiff in bending, the second laminate is quasi-isotropic and the third laminate is stiff in torsion. Plate aspect ratio,  $L/c$ , is equal to 3. This data was used to compute  $\beta$  parameters; eigenvalues were then calculated. The values  $\lambda/\lambda_0$  are shown in the fourth column of Table 1.

TABLE 1. LAMINATE STIFFNESS/DIVERGENCE CHARACTERISTICS

LAMINATE	$D_{11}/D_{66}$	$\beta$	$2\lambda/\pi$
$[0_2/\pm 30^\circ]_s$	16.94	0.0392	1.265
$[0/\pm 45/90]_s$	5.59	0.0129	1.14
$[\pm 45/\pm 45]_s$	1.216	0.0028	1.059

The final column of Table 1 shows the ratio of the eigenvalue  $\lambda$  (divergence speed is proportional to  $\lambda$ ) divided by  $\pi/2$ . This value,  $\pi/2$ , is the lowest value of  $\lambda$  that would be computed if  $S_0$  (or  $\beta$ ) were zero. Thus, for these three laminates, the inclusion of plate-like restraint of warping effects results in higher airspeeds being predicted for divergence than if  $\beta$  were zero. For the three laminates, the two predictions (with  $L/c = 3$ ) differ by 26.5%, 14% and 6.9%, respectively. To bring the 26.5% error down to 6.9%, the first laminate would have to have its aspect ratio increased from 3 to 10.8!

The mode shape at divergence is

$$\begin{aligned}\theta(\eta) = & A_1(\sin(g\eta) - f\sinh(f\eta)) \\ & + A_2(\cos(g\eta) - \cosh(f\eta))\end{aligned}\quad (22)$$

From boundary conditions, the constant  $A_2$  is:

$$A_2 = \frac{-A_1(g^2 \sin(g) + g f \sinh(f))}{(g^2 \cos g + f^2 \cosh f)} \quad (23)$$

$$\bar{\theta}(\eta) = \frac{\theta(\eta)}{\theta(1)} \quad (24)$$

If restraint of warping effects are excluded, this mode shape would simply be

$$\theta(\eta) = A \sin\left(\frac{\pi}{2} \eta\right) \quad (25)$$

The mode shapes or torsional deformation at divergence are both interesting and informative. Figure 3 shows the normalized mode shape given in Eqns. 22 - 24 for several values of  $\beta$ . Large values of  $\beta$  are associated either with low aspect ratio plates or with large values of  $D_{11}/D_{66}$ . Also shown is the classical St. Venant theory divergence mode shape ( $\beta = 0$ ). Even for small values of  $\beta$ , the mode shape at divergence is significantly different than the quarter-sine wave associated with St. Venant torsion theory. Differences are particularly noticeable in the region near  $\eta = 0$ . The idealization used in this study predicts airloads that are proportional to the mode shape for  $\theta(\eta)$  plotted in Figure 3. The increased divergence dynamic pressure that results when restraint of warping is considered is seen to be due to reduced incidence in the inboard region. When  $\beta$  equals unity there is a reversal of incidence in the region  $0 \leq \eta < 0.75$  which leads to very high divergence airspeeds.

#### 5.5.4 Conclusions

This study has examined the aeroelastic divergence of flat cantilever orthotropic plates. These results are compared to those obtained using classical St. Venant torsion theory that neglects plate bending terms that reflect restraint of warping of plate cross-sections. A non-dimensional parameter  $\beta$  is defined and used to determine the degree to which warping effects are important. This parameter is composed of the ratio of spanwise bending stiffness to torsional stiffness and plate structural aspect ratio. For values of  $\beta > 0.01$ , the inclusion of warping restraint is found to result in larger dynamic pressures at divergence than would be found using St. Venant torsion theory. Thus St. Venant torsion theory leads to overly conservative results. Equally important, for  $\beta > 0.01$  the divergence mode shape with restraint of warping included may be substantially different from that found using St. Venant torsion theory. This latter observation is important to the selection of assumed modes for approximate solutions to aeroelastic stability problems.

The differences encountered in this divergence study are due to the orthotropic behavior of plates and are not confined to laminated composites. However, some laminate designs display an additional feature known as bend/twist stiffness cross-coupling. This effect couples together the twisting and out-of-plane bending of the plate in such a fashion that the full set of equations, described earlier, must be solved. A second limitation of the present results is that camber bending has been suppressed by truncation of the series given in Eqn. 8. Inclusion of additional terms may be necessary to further refine the solution.

#### 5.5.5 References

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- [2] E.F. Crawley and J. Dugundji, "Frequency Determination and Non-dimensionalization for Composite Cantilever Plates," Journal of Sound and Vibration, Vol. 72, No. 1, pp 1-10, 1980.
- [3] G.A. Oyibo and J.H. Berman, "Influence of Warpage on Composite Aeroelastic Theories," AIAA Paper No. 85-0710, April 1985.

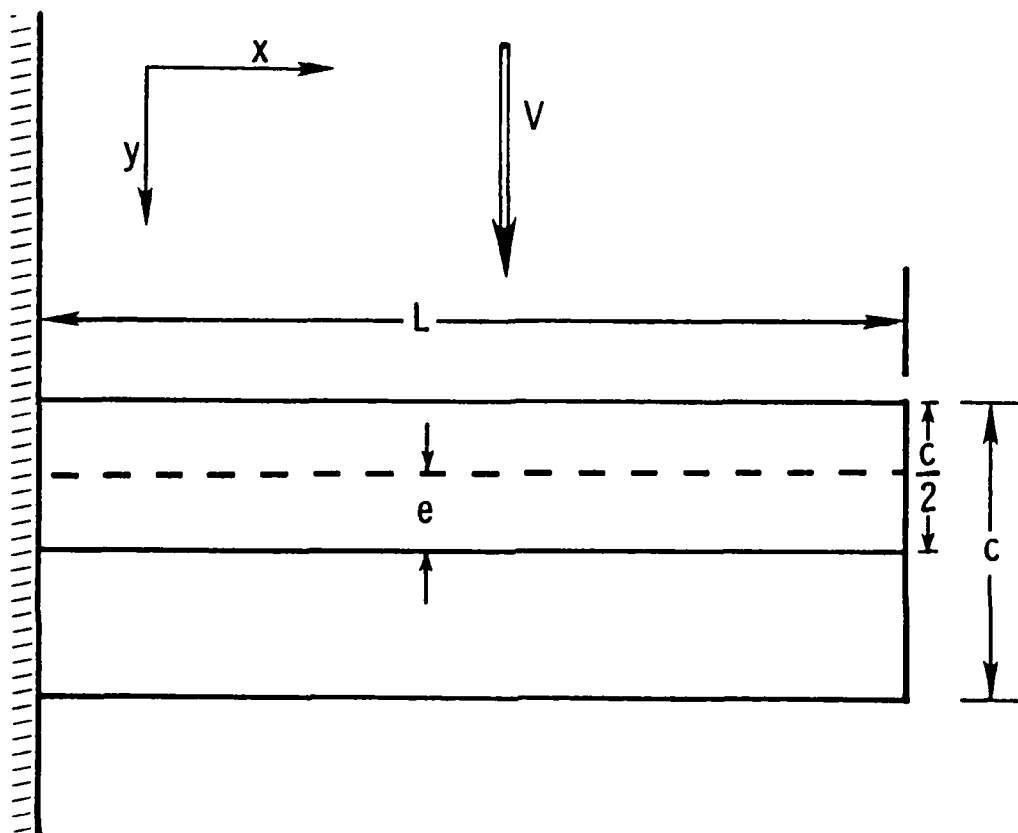


Figure 1. Idealized plate planform geometry

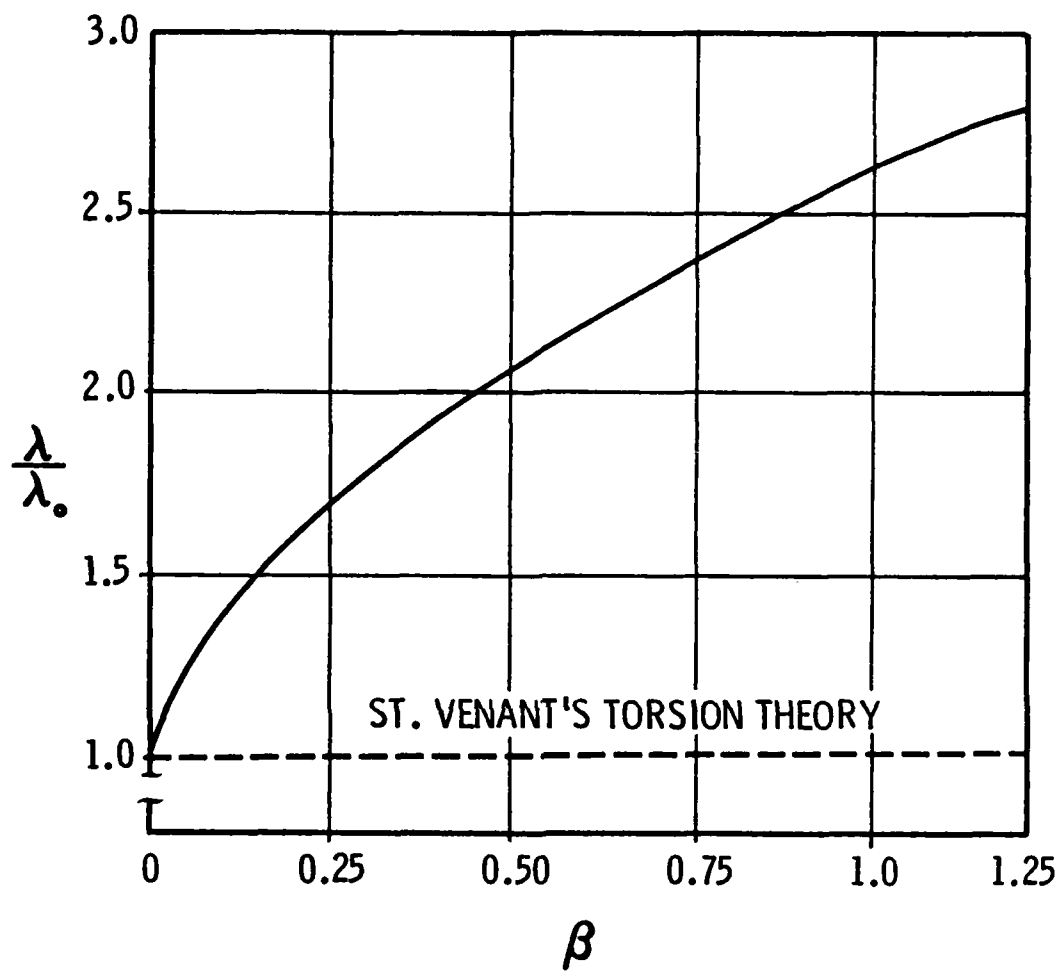


Figure 2. Non-dimensional airspeed parameter  $\lambda/\lambda_0$  versus warping parameter,  $\beta$

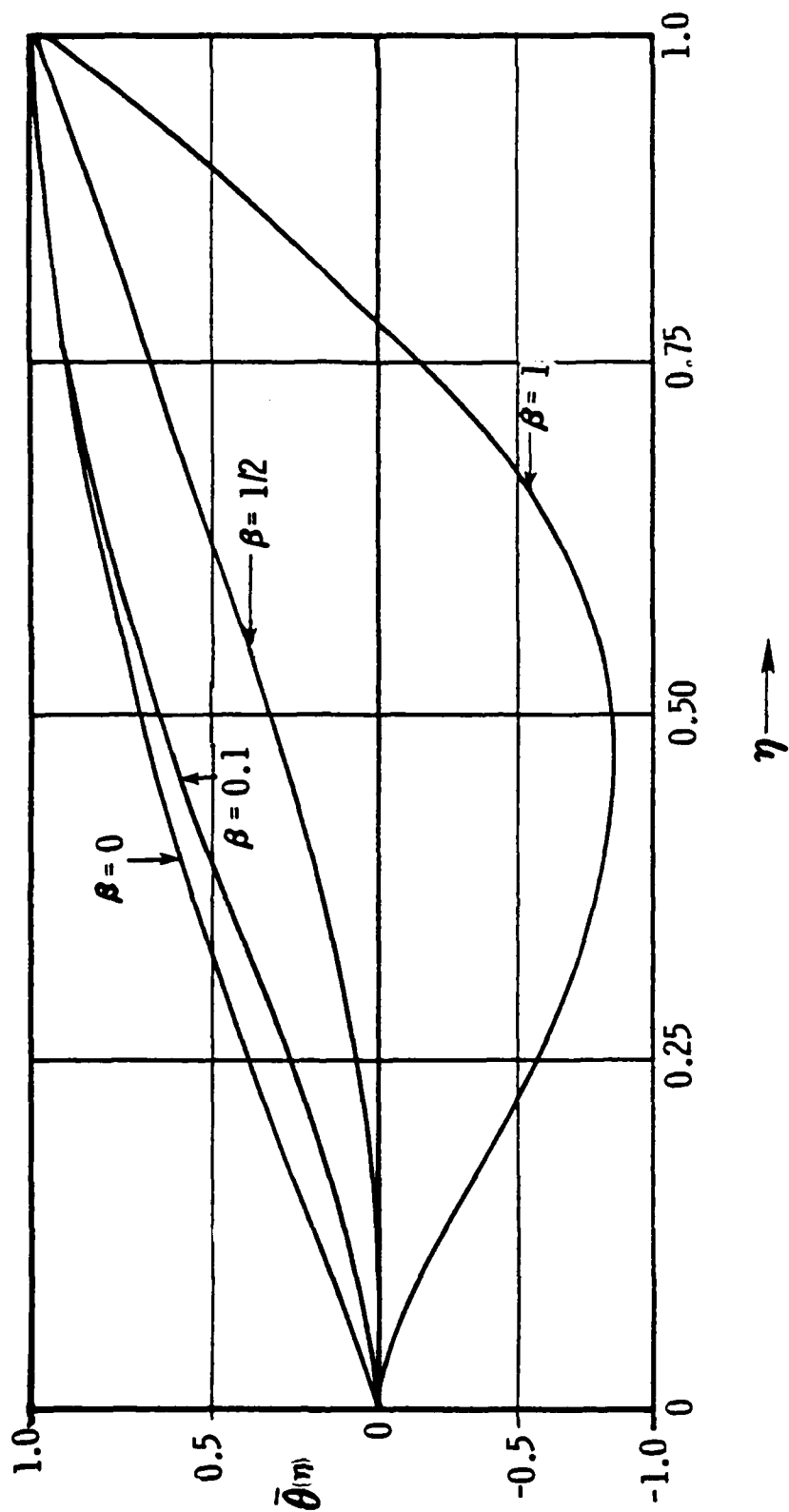


Figure 3. Divergence mode shape  $\bar{\theta}(\eta)$  for several values of  $\beta$

## 5.6 A STUDY OF THE EFFECTS OF PLATE STRUCTURAL IDEALIZATIONS UPON TAILORING METHODOLOGY by Terrence A. Weisshaar

The great wealth of analytical studies related to aeroelastic tailoring that have appeared recently have added insight and confidence to the advanced composite design integration process. Many different structural idealizations are used to demonstrate tailoring features and potential improvements to be found through tailoring. Some of these idealizations, while accurate for conventional metallics, have been found to be lacking in essential features when advanced composite construction is the mode of construction. Elimination or disregard of some elastic effects that may safely be neglected for metallics has been found to be unwise in the study of tailored structures.

The study of the effects of tailoring as it affects structural or aeroelastic dynamic response is conducted along two parallel approaches. The first approach is to select a certain geometric design, for instance a fixed planform, and to choose a specific material, e.g. Graphite/epoxy. The tailoring study then examines the effects of ply orientation, thickness and stacking sequence upon the response of interest. The results are valid only for that material and geometry and stacking sequence/thickness combination.

A second approach is to define a set of "natural variables" that are independent of any actual planform, material or geometry. Studies then identify desirable combinations of these natural variables and assess their effects upon response. Such an approach has advantages and disadvantages. A major advantage is that special behavior can be attributed to a distinct generic feature of laminated composites. When lam-

inate parameters are used, for instance, in a dimensional analysis that uses fiber sweep as a variable, a number of important stiffness variables change simultaneously. One is hard pressed to attribute desirable (or undesirable) changes to a specific cause. The use of natural variables that are nondimensional in nature eliminates this difficulty.

There are two, in particular, problems with the use of natural variables. First of all, how many truly independent nondimensional parameters are present? Secondly, having identified these parameters, what are their true physical limits or ranges of value? Both questions are important since any analytical model consists only of numbers and, if used incorrectly, may model an impossible physical process. A case in point is the insertion of a negative stiffness into a vibration equation or use of a negative viscous damping constant.

This study exams the problems associated with nondimensional modeling of plate vibration idealizations. The first section examines the circumstances under which the elements of the matrix of plate stiffnesses may be considered to be independent. The second section suggests nondimensional parameters for use in these types of studies and defines their number and limits. The third section illustrates the use of these parameters in a free vibration study.

#### 5.6.1 Plate Stiffness Parameter Independence for Tailoring Studies

Lifting surface structural idealizations for tailoring studies frequently involve the use of equivalent plate assumptions. In this case, there are six plate constants ( $D_{11}$ ,  $D_{12}$ ,  $D_{22}$ ,  $D_{26}$ ,  $D_{16}$ ,  $D_{66}$ ) to be determined. The effects of changes in these six parameters upon free

vibration and response to external deformation dependent loads is of importance to design optimization. However, before any parameter variation can be studied, it must be determined which plate stiffness parameters are truly independent and under what conditions or restrictions this parametric independence arises. An examination of this determination is the subject of this section.

To become familiar with the most basic situation, first consider the isotropic plate stiffnesses. These are:

$$D_{11} = D_{22} = D = Et^3/12(1 - \mu^2) \quad (1.1)$$

$$D_{16} = D_{26} = 0 \quad (1.2)$$

$$D_{12} = \mu D \quad (1.3)$$

$$D_{66} = D(1 - \mu)/2 \quad (1.4)$$

Isotropic plate design uses three independent design variables,  $E$ ,  $t$  and  $\mu$ . Material constitutive relationships do not allow  $D_{11}$  and  $D_{22}$  to be changed independently. It would appear at first glance that three of the plate stiffnesses could be treated as independent, but this is not the case since  $E$  and  $t^3$  appear as a unit. Let us specify that  $D_{11} = \bar{D}$  and  $D_{66} = \bar{G}$ , where  $\bar{D}$  and  $\bar{G}$  are fixed values. Solving for the parameters  $E$ ,  $t$ ,  $\mu$ , to construct such a plate, we find:

$$\mu = 1 - 2\frac{D_{66}}{D_{11}} = 1 - 2\frac{\bar{G}}{\bar{D}} \quad (1.5)$$

$$Et^3 = 48D_{66}\left(1 - \frac{D_{66}}{D_{11}}\right) = 48\bar{G}\left(1 - \frac{\bar{G}}{\bar{D}}\right) \quad (1.6)$$

$D_{12}$  is now fixed to be:

$$D_{12} = \bar{D}\left(1 - 2\frac{\bar{G}}{\bar{D}}\right) \quad (1.7)$$

Note also that the equation for  $\mu$  has limits  $-1 < \mu < 0.5$  for isotropic

materials so that the physical limits on choices of  $\frac{\bar{G}}{D}$  are:

$$\frac{1}{4} < \frac{\bar{G}}{D} < 1 \quad (1.8)$$

This means that physically we cannot construct a plate with stiffness ratio  $D_{66}/D_{11}$  outside these limits if we are restricted to isotropic materials. A question remains, if there are three design parameters ( $E, \nu, \mu$ ) why can't we arbitrarily fix three plate constants? This question will be addressed later in the discussion.

This example indicates that two plate stiffnesses are independent while the third stiffness depends upon the other two because of material constitutive relations. We also see that there is only one independent ratio of stiffnesses, that is, either

$$\frac{D_{22}}{D_{11}} = 1 - 2\frac{D_{66}}{D_{11}} \quad (1.9)$$

$$\text{or } \frac{D_{66}}{D_{11}} = 1 - 2\frac{D_{12}}{D_{11}} \quad (1.10)$$

Next, let us consider an orthotropic material for which

$$D_{11} = \frac{Q_{11}t^3}{12} \quad (1.11)$$

$$D_{12} = \frac{Q_{12}t^3}{12} \quad (1.12)$$

$$D_{22} = \frac{Q_{22}t^3}{12} \quad (1.13)$$

$$D_{66} = \frac{Q_{66}t^3}{12} \quad (1.14)$$

$$D_{16} = D_{26} = 0 \quad (1.15)$$

Note that  $t^3$  appears in every  $D_{ij}$  parameter and it cannot be used as an independent design parameter to adjust relative values of the  $D_{ij}$ 's.

For the orthotropic plate there are four material parameters,  $Q_{11}$ ,  $Q_{12}$ ,  $Q_{22}$ , and  $Q_{66}$ . If these material parameters are independent, then all four plate stiffnesses may be prescribed independently. Consider the case for which

$$Q_{11} = E_{11}/(1 - \mu_{12}\mu_{21}) \quad (1.16)$$

$$Q_{22} = E_{22}/(1 - \mu_{12}\mu_{21}) \quad (1.17)$$

$$\begin{aligned} Q_{12} &= \mu_{12}E_{22}/(1 - \mu_{12}\mu_{21}) \\ &= \mu_{21}E_{11}/(1 - \mu_{12}\mu_{21}) \end{aligned} \quad (1.18)$$

$$Q_{66} = G_{12} \quad (1.19)$$

Let  $\bar{E}_{11} = E_{11}t^3$ ,  $\bar{E}_{22} = E_{22}t^3$ ,  $\bar{G}_{12} = G_{12}t^3$  with  $t^3$  fixed. If  $\bar{D}_{11}$ ,  $\bar{D}_{22}$ ,  $\bar{D}_{12}$  and  $\bar{D}_{66}$  are the prescribed values of plate stiffness, then the values of  $\bar{E}_{11}$ ,  $\bar{E}_{22}$ ,  $\mu_{12}$  (or  $\mu_{21}$ ) and  $\bar{G}_{12}$  necessary to guarantee this selection are:

$$\bar{E}_{11} = 12\bar{D}_{11}(1 - \bar{D}_{12}^2/\bar{D}_{11}\bar{D}_{22}) \quad (1.20)$$

$$\bar{E}_{22} = 12\bar{D}_{22}(1 - \bar{D}_{12}^2/\bar{D}_{11}\bar{D}_{22}) \quad (1.21)$$

$$\mu_{12} = \bar{D}_{12}/\bar{D}_{22} \quad \text{or} \quad \mu_{21} = \bar{D}_{12}/\bar{D}_{11} \quad (1.22)$$

$$\bar{G}_{12} = 12\bar{D}_{66} \quad (1.23)$$

Therefore, to the extent that materials can be constructed according to these latter specifications, the four plate stiffness parameters are independent. Notice that the nondimensional ratio  $\bar{D}_{12}^2/\bar{D}_{11}\bar{D}_{22}$  appears in two of these equations. If we were to require values of  $\bar{D}_{12}$ ,  $\bar{D}_{11}$ ,  $\bar{D}_{22}$  that led to values of this ratio in excess of unity, then  $\bar{E}_{11}$  and  $\bar{E}_{22}$  would be negative. Such cases are prohibited by thermodynamical consideration, namely energy conservation. Additional study of the way in

which the plate stiffness parameters may be varied is in order.

The availability of laminated composite materials introduces design freedom to the materials selection process. Rather than constructing a material with engineering constants  $E_{11}$ ,  $E_2$ ,  $\nu_{12}$ ,  $G_{12}$  it is more likely that a Hooke's Law or stiffness matrix  $[Q_{ij}]$  will be constructed. Discussion of this construction will be restricted to laminates with plies of equal thickness and a fixed overall thickness.

Let us first consider a single ply of thickness  $t$ . Plate stiffnesses are defined as:

$$D_{11} = \frac{\bar{Q}_{11} t^3}{12} \quad (1.24)$$

$$D_{12} = \frac{\bar{Q}_{12} t^3}{12} \quad (1.25)$$

$$D_{22} = \frac{\bar{Q}_{22} t^3}{12} \quad (1.26)$$

$$D_{16} = \frac{\bar{Q}_{16} t^3}{12} \quad (1.27)$$

$$D_{26} = \frac{\bar{Q}_{26} t^3}{12} \quad (1.28)$$

$$D_{66} = \frac{\bar{Q}_{66} t^3}{12} \quad (1.29)$$

The bar over the  $\bar{Q}_{ij}$  terms indicates that they have been transformed by a ply angle rotation. The  $\bar{Q}_{ij}$  quantities are defined in terms of material properties and angular rotation  $\theta$  (Ref. 1).

$$\bar{Q}_{11} = U_1 + U_2 \cos(2\theta) + U_3 \cos(4\theta) \quad (1.30)$$

$$\bar{Q}_{22} = U_1 - U_2 \cos(2\theta) + U_3 \cos(4\theta) \quad (1.31)$$

$$\bar{Q}_{12} = U_4 - U_3 \cos(4\theta) \quad (1.32)$$

$$\bar{Q}_{66} = U_5 - U_3 \cos(4\theta) \quad (1.33)$$

$$\bar{Q}_{16} = -\frac{1}{2} U_2 \sin(2\theta) - U_3 \sin(4\theta) \quad (1.34)$$

$$\bar{Q}_{26} = -\frac{1}{2}U_2 \sin(2\theta) + U_3 \sin(4\theta) \quad (1.35)$$

The  $U_i$  terms are combinations of ply material properties. They are written as:

$$U_1 = \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}) \quad (1.36)$$

$$U_2 = \frac{1}{2}(Q_{11} - Q_{22}) \quad (1.37)$$

$$U_3 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}) \quad (1.38)$$

$$U_4 = \frac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}) \quad (1.39)$$

$$U_5 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}) = \frac{1}{2}(U_1 - U_4) \quad (1.40)$$

Four of the five  $U_i$  terms are independent and invariant for a specified material. Material selection to define the  $U_i$ 's is analogous to choosing  $E$  and  $\mu$  for an isotropic material. This material choice constitutes a design freedom. Another design freedom is ply orientation,  $\theta$ . Once the single material is prescribed, for a laminated material only four of the six  $D_{ij}$ 's may be chosen independently. Combining equations (1.24-1.40) we have:

$$D_{11} + D_{22} + 2D_{12} = 2(U_1 + U_4)\frac{t^3}{12} \quad (1.41)$$

$$\text{and } D_{66} - D_{12} = (U_5 - U_4)\frac{t^3}{12} \quad (1.42)$$

Leaving the case of the single ply, next consider a laminate with 4 plies of a given material. Restrict the discussion to symmetrical laminates. Tsai and Hahn (Ref. 1) show that the plate stiffnesses may be written as:

$$D_{11} = U_1 V_{OD} + U_2 V_{ID} + U_3 V_{3D} \quad (1.43)$$

$$D_{12} = U_4 V_{OD} - U_3 V_{3D} \quad (1.44)$$

$$D_{22} = U_1 V_{OD} - U_2 V_{ID} + U_3 V_{3D} \quad (1.45)$$

$$D_{16} = -\frac{1}{2}U_2 V_{2D} + U_3 V_{4D} \quad (1.46)$$

$$D_{26} = -\frac{1}{2}U_2 V_{2D} + U_3 V_{4D} \quad (1.47)$$

$$D_{66} = U_5 V_{OD} - U_3 V_{3D} \quad (1.48)$$

where  $V_{OD} = t^3/12$ .

The terms  $V_{1D}$ ,  $V_{2D}$ ,  $V_{3D}$ ,  $V_{4D}$  are defined, for a laminate with plies,  $2n$  symmetrical.

$$V_{1D} = 2 \sum_{k=1}^n \sin(2\theta_k) \beta_k \quad (1.49)$$

$$V_{2D} = 2 \sum_{k=1}^n \sin(2\theta_k) \beta_k \quad (1.50)$$

$$V_{3D} = 2 \sum_{k=1}^n \cos(4\theta_k) \beta_k \quad (1.51)$$

$$V_{4D} = 2 \sum_{k=1}^n \sin(4\theta_k) \beta_k \quad (1.52)$$

The terms  $\beta_k$  are equal to  $\frac{1}{3}(t_k^3 - t_{k-1}^3)$  and correspond to area moments of inertia of the  $k$ th plies ( $1 < k < n$ ) lying between positions  $t_{k-1}$  and  $t_k$  from the plate midsurface.

With 4 plies (2 independent layers), a given material and with  $t^3$  fixed ( $V_{OD}$  is then prescribed) only two of the values of  $D_{ij}$  are independent. This is so because Eqns. 1.41 and 1.42 are valid no matter how many plies are used. Let's choose  $D_{16}$  and  $D_{26}$  to be zero. In this case,

$$+\frac{1}{2}U_2 V_{2D} = -U_3 V_{4D} \quad (1.53)$$

and

$$\frac{1}{2}U_2 V_{2D} = U_3 V_{4D} \quad (1.54)$$

The only solution in this case is  $V_{2D} = V_{4D} = 0$ ; For this situation,  $\theta_k = 0, \pm 90^\circ$ , corresponding to a unidirectional laminate  $[0^\circ, 0^\circ]_s$  or a cross-ply laminate  $[0^\circ, \pm 90^\circ]_s$ . The parameters  $D_{11}$ ,  $D_{12}$ ,  $D_{22}$ , and  $D_{66}$

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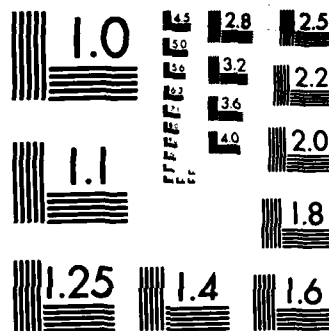
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are now determined since no other design variables remain.

Adding a third ply (6 layers) implies that three  $D_{ij}$  values are independently "tailorable" since three of the  $V$  summations in Eqns. 1.49-1.52 can be controlled. For instance, we might choose  $D_{16} = D_{26} = 0$  and  $D_{66} = \bar{D}_{66}$ . In principle three values of  $\theta_k$  could be found to create this situation (note that sine and cosine are periodic). Extending this discussion, four plies allow the choice of all four values of  $V$  and the four independent  $D_{ij}$ 's can be specified.

What happens when the number of plies exceeds four? In this case, the number of design variables exceeds the number of  $V_{01}$  parameters. There are more unknowns ( $\theta_k$ 's) than equations. The system is overdetermined and thus the  $\theta_k$  solution set to fix  $V_{1D}$ ,  $V_{2D}$ ,  $V_{3D}$ ,  $V_{4D}$  is not unique. This is one great advantage of advanced composites, since there are a number of different laminate designs with equivalent stiffnesses.

#### 5.6.2 Conclusion

It appears that the six plate stiffness parameters ( $D_{ij}$ ) can be regarded as independent under certain conditions. For instance, if a certain ply material is chosen then four or more plies provide tailoring independence for four plate stiffnesses, within the limits imposed by material relations. Choosing the material properties then adds additional freedom necessary to make the remaining two plate stiffnesses independent. If the  $D_{ij}$  parameters are to be regarded as independent then one must allow changes in material.

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### 5.6.3 Appendix

The suppression of camber bending in plate vibrations can lead to serious errors in both the magnitude of the natural frequency and the mode shape determined from the equilibrium equation. In unpublished work by Liu (not related to the present contract) he compares the plate frequency with the results using the classical beam bending frequency expression and the classical beam torsion equation. His comparisons extend over a wide range of aspect ratios, but are restricted to orthotropic plates. A summary of these results follows.

A rectangular, cantilever plate with the following dimensions is considered. The chord dimension is  $c=20''$  while the span dimension is  $L = rc = 20r$ , where  $r$  is the plate aspect ratio. Plate stiffnesses are fixed to be:

$$\begin{aligned} D_{11} &= 122.92 \text{ lb-in.} & D_{16} &= D_{26} = 0 \\ D_{12} &= 33.66 \text{ lb-in.} & D_{66} &= 46.5 \text{ lb-in.} \\ D_{22} &= 801.59 \text{ lb-in.} \end{aligned}$$

The mass per unit area is  $\rho = 0.00171 \text{ slug/in}^2$ . For comparison, beam parameters are defined as:

$$EI = c(D_{22} - D_{12}^2/D_{11})$$

$$M = \rho c$$

The frequencies of the first two bending modes are predicted to be:

$$f_{1b} = 1319.6/L^2 \quad \text{and} \quad f_{2b} = 8269.5/L^2 \text{ (Hz.)}$$

For torsional vibration, with  $t$  = plate thickness,

$$GJ = 4cD_{66}$$

$$J = \frac{1}{12}\rho c^3(1 + (t/c)^2) = \frac{1}{12}\rho c^3$$

where the ratio  $t/c$  has been neglected. The first two torsional frequencies are predicted to be:

$$f_{1t} = 49.47/L \text{ (Hz.)}$$

$$f_{2t} = 148.4/L \text{ (Hz.)}$$

A Rayleigh-Ritz analysis was completed for 12 different values of aspect ratio,  $r$ . A polynomial series representation was used with 8 terms in the spanwise direction. In the chordwise direction the number of polynomial terms (denoted as  $n_x$ ) was varied. If  $n_x = 1$  then chordwise rigidity is assumed. If  $n_x = 2$  parabolic camber bending is allowed. Results are shown on the following pages. In each case, columns two and three represent the ratio of natural frequency predicted from Rayleigh-Ritz plate analysis to natural frequency predicted from elementary beam analysis.

These results illustrate two effects. As shown by previous authors, torsional frequency error is strongly dependent upon aspect ratio. This is true whether camber bending is neglected or not. This difference between plate results and beam results is due to the neglect of inplane warping in the beam analysis. In the case of the bending frequency, as aspect ratio increases, the discrepancy between plate theory and bending theory actually increases if camber bending is suppressed. The discrepancy is diminished with increasing aspect ratio of camber bending ( $n_x = 2$ ) is permitted.

# First Bending Mode

## Frequency Ratio

r	$n_x = 1$	$n_x = 2$
0.5	1.02495	1.00622
1	1.05045	1.00580
1.5	1.07180	1.00525
2	1.08865	1.00470
2.5	1.10202	1.00422
3	1.11284	1.00382
6	1.14928	1.00240
9	1.16499	1.00196
12	1.17372	1.00175
15	1.17940	1.00205
18	1.18271	1.00196
21	1.18583	1.00134

# Second Bending Mode

## Frequency Ratio

r	$n_x = 1$	$n_x = 2$
0.5	1.00505	1.00628
1	1.00544	1.00676
1.5	1.00602	0.99940
2	1.00655	1.00282
2.5	1.00701	1.00294
3	1.00741	1.00285
6	1.00886	1.00218
9	1.00944	1.00188
12	1.00982	1.00174
15	1.01012	1.00174
18	1.01034	1.00157
21	1.01045	1.00149

# First Torsion Mode

## Frequency Ratio

r	$n_x = 1$	$n_x = 2$
0.5	3.18069	3.13645
1	2.3645	1.88918
1.5	1.68532	1.56957
2	1.51105	1.41890
2.5	1.40401	1.32903
3	1.33102	1.26886
6	1.14676	1.12157
9	.09129	1.07695
12	1.06535	1.05632
15	1.05094	1.04475
18	1.04199	1.03755
21	1.03608	1.02269

# Second Torsion Mode

## Frequency Ratio

r	$n_x = 1$	$n_x = 2$
0.5	5.76029	5.73608
1	3.07744	3.05159
1.5	2.24352	2.22044
2	1.85843	1.83864
2.5	1.64394	1.62703
3	1.50946	1.49490
6	1.20643	1.19933
9	1.11993	1.11573
12	1.08204	1.07929
15	1.06189	1.05991
18	1.04977	1.04832
21	1.04194	1.04081

#### 5.7 AN EXTENDED PLATE/BEAM FINITE ELEMENT FOR COMPOSITE TAILORING STUDIES by Terrence A. Weisshaar

There is a growing appreciation of the importance of including inplane shear stiffness effects into analytical models that represent slender, advanced composite, beam-like structural elements. This is the so-called "restraint of warping effect" discussed in a number of recent studies (1-3). This effect stems from constraining the root cross-section of a plate-like or beam-like structure to remain undeformed in its own plane, an effect that is usually unaccounted for in elementary studies. Accountability for the warping constraint may be handled with several levels of mathematical complexity. In the simplest case, one might simply resort to a detailed finite element model of an anisotropic plate-like lifting surface. Such an approach is cumbersome and may be found, after great effort, to be unnecessary. On the other hand, restraint of warping effects are automatically included in any plate analysis.

An alternative way of retaining accuracy while maintaining computational simplicity is to develop an "extended beam" finite element. By extended, it is meant that an important plate-like feature, that of chordwise-camber, is retained as a nodal degree-of-freedom to be either used or suppressed. Such elements, with limitations, are discussed by Weisshaar [4] and Chen and Yang [5]. Weisshaar's model is deficient in that it does not include transverse shear deformation, nor does it include sophisticated shape functions such as those used by Chen and Yang. Chen and Yang, on the other hand, do not retain camber

bending as a degree of freedom. As a result, they exclude an important effect from an otherwise useful element.

The term "extended beam-element", as applied to composite tailoring studies, denotes a modified beam idealization for which at least parabolic, chordise camber (bending) is retained. Otherwise, torsional stiffness as well as mode shape information is flawed. As a result, the accuracy of aeroelastic stability studies may be compromised.

This appendix outlines the basic steps necessary to develop an extended beam finite element for dynamical studies. Such an element is essential for checking the results of other, more sophisticated, studies as well as generating useful trend information.

Consider the beam element shown in Figure A-1. The element has three degrees of freedom at each end. These are: (1) bending displacement; (2) bending slope; (3) torsional twist. A beam cantilever condition will require that all three degrees of freedom be zero. A plate cantilever condition requires that the edge (perpendicular to the plane of the paper) remain straight. This translates into the requirement that the twist rate (first derivative of torsional twist) be zero and that no camber due to Poisson's ratio effects be present. For the model shown in Figure A-1, such constraints are uncontrollable.

To control boundary conditions related to camber and warping restraints, two additional degrees of freedom at each node are required. One degree of freedom is twist rate while the second degree of freedom is camber curvature. Let us turn to development of the theoretical expressions necessary to define an extended beam finite element stiffness matrix.

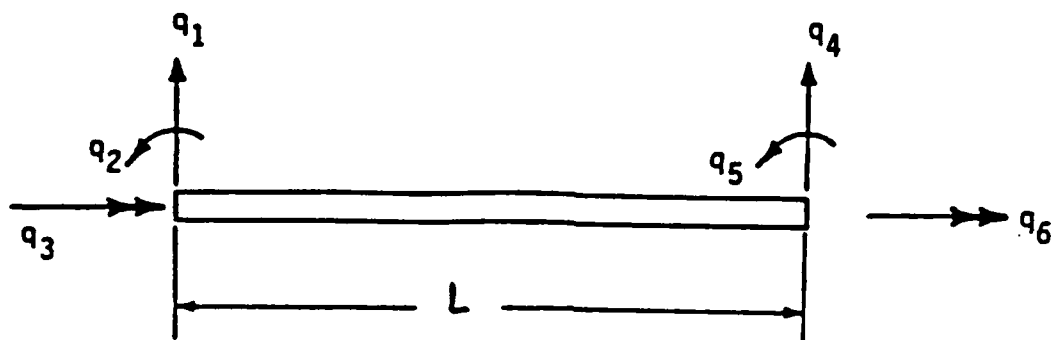


Figure A-1 Six degree-of-freedom beam element

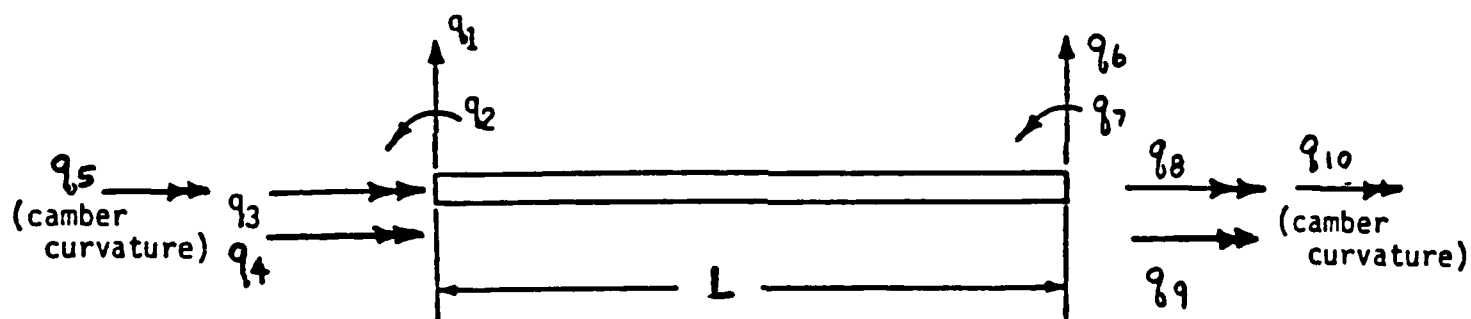


Figure A-2 Ten degree-of-freedom extended beam element

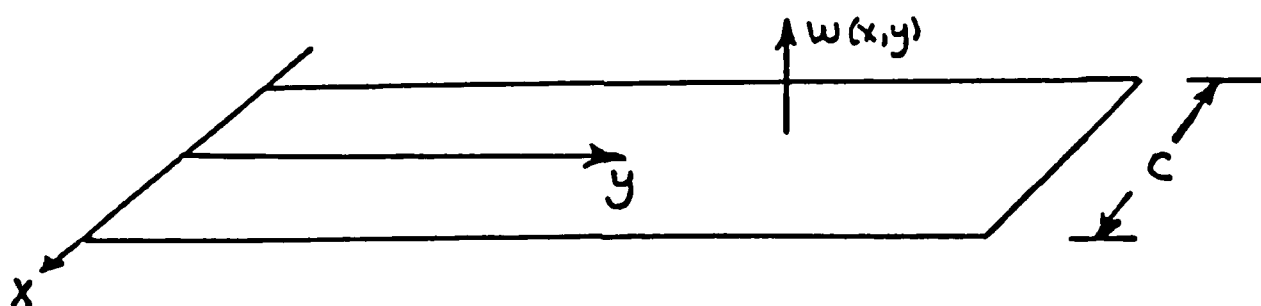


Figure A-3 Plate coordinate system

Shown in Figure A-2 is an element of length  $L$  with five degrees of freedom at each end. These are, at the ends: (1) and (6), bending displacement; (2) and (7), bending slope; (3) and (8), twist rotation; (4) and (9), twist rate; and, (5) and (10), camber curvature. An assumed displacement function,  $w(x,y)$ , is defined to be (positive upward)

$$w(x,y) = \left\{ \gamma_i(y) \right\} - x \left\{ \delta_i(y) \right\} \{q_i\} + x^2 \left\{ \beta_i(y) \right\} \{q_i\} \quad (A.1)$$

In Eqn. A.1 the functions  $\gamma_i$ ,  $\delta_i$ ,  $\beta_i$  are functions of the spanwise coordinate,  $y$ . The functions  $\gamma_i$  provide spanwise displacement due only to bending, while the  $\delta_i$  functions give information on twisting effects. The  $\beta_i$  functions account for parabolic, chordwise camber effects.

Let us first consider a beam model based upon laminated plate theory. In the case of a mid-surface symmetric laminated plate, the relationship between plate bending moments, twisting moment and curvatures may be expressed as (see for instance, Tsai and Hahn):

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (A.2)$$

The elements  $D_{ij}$  are anisotropic plate stiffness constants that are functions of laminate ply geometry, material properties and stacking sequence, while  $\kappa_x$ ,  $\kappa_y$  and  $\kappa_{xy}$  are plate curvatures.

If we adopt the  $y$ -axis in Figure A-3 as a reference axis, the plate deflection  $w(x,y)$  and the beam deflection  $h(y)$  and twist  $\alpha(y)$  are defined as:

$$h(y) = w(0,y) \quad (A.3)$$

and

$$\frac{\partial w}{\partial x} \Big|_{x=0} = -a(y) \quad (A.4)$$

In this case, the plate curvatures can be approximated as follows,

$$\kappa_y = \partial^2 w / \partial y^2 \quad (A.5)$$

and

$$\kappa_x = \partial^2 w / \partial x^2 \quad (A.6)$$

$$\kappa_{xy} = 2 \frac{\partial^2 w}{\partial x \partial y} \quad (A.7)$$

The strain energy density of the plate is written as

$$U_o = \frac{1}{2} \begin{bmatrix} \kappa_x & \kappa_y & \kappa_{xy} \end{bmatrix} [D_{ij}] \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (A.8)$$

Now, let

$$\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = [\Lambda] \{q\} \quad (A.9)$$

where

$$[\Lambda] = \begin{bmatrix} 2\beta_1 \\ \gamma_1'' - x\delta_1'' + x^2\beta_1'' \\ -2\delta_1' + 2x\beta_1' \end{bmatrix} \quad (A.10)$$

and the notation  $()'$  denotes differentiation with respect to  $y$ .

As a result, the system stiffness matrix is given by the expression:

$$[K_{ij}] = \int_{-c/2}^{c/2} \int_0^L [\Lambda]^T [D_{ij}] [\Lambda] dx dy \quad (A.11)$$

The elemental stiffness matrix depends upon the choice of assumed displacement shape functions  $\gamma_1, \delta_1, \beta_1$ . For  $\gamma_1$ , choose

$$\begin{aligned} \gamma_1 &= 1 - 3\left(\frac{y}{L}\right)^2 + 2\left(\frac{y}{L}\right)^3 \\ \gamma_2 &= -y + 2y^2/L - y^3/L^2 \\ \gamma_3 &= \gamma_4 = \gamma_5 = 0 \\ \gamma_6 &= 3\left(\frac{y}{L}\right)^2 - 2\left(\frac{y}{L}\right)^3 \\ \gamma_7 &= -y^3/L^2 + y^2/L \\ \gamma_8 &= \gamma_9 = \gamma_{10} = 0 \end{aligned}$$

For an approximation to elemental twist, choose

$$\begin{aligned} \delta_1 &= \delta_2 = 0 \\ \delta_3 &= \gamma_1 \\ \delta_4 &= \gamma_2 \\ \delta_5 &= 0 = \delta_6 = \delta_7 \\ \delta_8 &= \gamma_6 \\ \delta_9 &= \gamma_7 \\ \delta_{10} &= 0 \end{aligned}$$

For camber deformation, let

$$\begin{aligned} \beta_1 &= \beta_2 = \beta_3 = \beta_4 = 0 \\ \beta_5 &= 1 - 4\left(\frac{y}{L}\right) + 3\left(\frac{y}{L}\right)^2 \\ \beta_6 &= \beta_7 = \beta_8 = \beta_9 \\ \beta_{10} &= 3\left(\frac{y}{L}\right)^2 - 2\left(\frac{y}{L}\right) \end{aligned}$$

After multiplying the triple matrix product shown and integrating with respect to  $x$ , we have the following relationship:

$$K_{ij} = \int_0^L k_{ij} dy$$

The elements  $k_{ij}$ , which require integration, are (note  $k_{ij} = k_{ji}$ ):

$$\begin{aligned} k_{11} &= cD_{22}(\gamma_1'')^2 \\ k_{12} &= cD_{22}\gamma_1''\gamma_2'' \\ k_{13} &= -2cD_{26}\gamma_1''\gamma_1'' \\ k_{14} &= -2cD_{26}\gamma_1''\gamma_2'' \\ k_{15} &= 2cD_{12}\beta_5\gamma_1'' + \frac{c}{12}D_{22}\gamma_1''\beta_5'' \\ k_{16} &= cD_{22}\gamma_1''\gamma_6'' \\ k_{17} &= cD_{22}\gamma_1''\gamma_7'' \\ k_{18} &= -2cD_{26}\gamma_1''\gamma_6'' \\ k_{19} &= -2cD_{26}\gamma_1''\gamma_7'' \\ k_{1,10} &= 2cD_{12}\beta_{10}\gamma_1'' + \frac{c}{12}D_{22}\gamma_1''\beta_{10}'' \\ k_{22} &= cD_{22}(\gamma_2'')^2 \\ k_{23} &= -2cD_{26}\gamma_1''\gamma_2'' \\ k_{24} &= -2cD_{26}\gamma_2''\gamma_2'' \\ k_{25} &= \frac{c}{12}D_{22}\beta_5\gamma_2'' + 2cD_{12}\beta_5\gamma_2'' \\ k_{26} &= cD_{22}\gamma_2''\gamma_6'' \\ k_{27} &= cD_{22}\gamma_2''\gamma_7'' \\ k_{28} &= -2cD_{26}\gamma_2''\gamma_6'' \\ k_{29} &= -2cD_{26}\gamma_2''\gamma_7'' \\ k_{2,10} &= 2cD_{12}\beta_{10}\gamma_2'' + \frac{c}{12}D_{22}\beta_{10}\gamma_2'' \\ k_{33} &= \frac{c}{12}D_{22}(\gamma_1'')^2 + 4cD_{66}(\gamma_1')^2 \\ k_{34} &= \frac{c}{12}D_{22}\gamma_1''\gamma_2'' + 4cD_{66}\gamma_1'\gamma_2' \end{aligned}$$

$$\begin{aligned}
k_{35} &= -2 \frac{c^3}{12} D_{26} [\gamma_1'' \beta_5' + \gamma_1' \beta_5''] - 4cD_{16} \beta_5 \gamma_1' \\
k_{36} &= -2cD_{26} \gamma_1' \gamma_6'' \\
k_{37} &= -2cD_{26} \gamma_1' \gamma_7'' \\
k_{38} &= \frac{c^3}{12} D_{22} \gamma_1'' \gamma_6'' + 4cD_{66} \gamma_1' \gamma_6' \\
k_{39} &= \frac{c^3}{12} D_{22} \gamma_1'' \gamma_7'' + 4cD_{66} \gamma_1' \gamma_7' \\
k_{3,10} &= -2D_{26} \frac{c^3}{12} [\beta_{10}' \gamma_1'' + \beta_{10}'' \gamma_1'] - 4cD_{16} \beta_{10} \gamma_1' \\
k_{44} &= \frac{c^3}{12} D_{22} (\gamma_2'')^2 + 4cD_{66} (\gamma_2')^2 \\
k_{45} &= -2D_{26} \frac{c^3}{12} [\beta_5' \gamma_2'' + \beta_5'' \gamma_2'] - 4cD_{16} \beta_5 \gamma_2' \\
k_{46} &= -2cD_{26} \gamma_2' \gamma_6'' \\
k_{47} &= -2cD_{26} \gamma_2' \gamma_7'' \\
k_{48} &= \frac{c^3}{12} D_{22} \gamma_2'' \gamma_6'' + 4cD_{66} \gamma_2' \gamma_6' \\
k_{49} &= \frac{c^3}{12} D_{22} \gamma_2'' \gamma_7'' + 4cD_{66} \gamma_2' \gamma_7' \\
k_{4,10} &= -2cD_{26} \left[ \frac{c^3}{12} \right] [\beta_{10}' \gamma_2'' + \beta_{10}'' \gamma_2'] - 4D_{16} c \beta_{10} \gamma_2' \\
k_{55} &= 4cD_{11} (\beta_5')^2 + \frac{4c^3}{12} D_{12} \beta_5 \beta_5'' \\
&\quad + \frac{c^5}{80} D_{22} (\beta_5'')^2 + 4D_{66} \frac{c^3}{12} (\beta_5')^2 \\
k_{56} &= 2cD_{12} \beta_5 \gamma_6'' + \frac{c^3}{12} D_{22} \gamma_6'' \beta_5' \\
k_{57} &= 2cD_{12} \beta_5 \gamma_7'' + \frac{c^3}{12} D_{22} \gamma_7'' \beta_5' \\
k_{58} &= -4cD_{16} \gamma_6' \beta_5 - \frac{2c^3}{12} D_{26} [\gamma_6' \beta_5'' + \beta_5' \gamma_6''] \\
k_{59} &= -4D_{16} \beta_5 \gamma_7' - 2D_{26} \frac{c^3}{12} [\gamma_7' \beta_5'' + \beta_5' \gamma_7''] \\
k_{5,10} &= 4cD_{11} \beta_5 \beta_{10} + 2D_{12} \frac{c^3}{12} [\beta_5 \beta_{10}'' + \beta_{10} \beta_5''] \\
&\quad + \frac{c^5}{80} D_{22} \beta_5'' \beta_{10}'' + 4cD_{66} \frac{c^3}{12} \beta_5' \beta_{10}'
\end{aligned}$$

$$\begin{aligned}
k_{66} &= cD_{22}(\gamma_6'')^2 \\
k_{67} &= cD_{22}\gamma_6''\gamma_7'' \\
k_{68} &= -2cD_{26}\gamma_6''\gamma_6''' \\
k_{69} &= -2cD_{26}\gamma_7''\gamma_6''' \\
k_{6,10} &= 2cD_{12}\beta_{10}'\gamma_6'' + \frac{c^3}{12}D_{22}\beta_{10}''\gamma_6''' \\
k_{77} &= cD_{22}(\gamma_7'')^2 \\
k_{78} &= -2cD_{26}\gamma_6''\gamma_7''' \\
k_{79} &= -2cD_{26}\gamma_7''\gamma_7''' \\
k_{7,10} &= 2cD_{12}\beta_{10}'\gamma_7'' + \frac{c^3}{12}D_{22}\beta_{10}''\gamma_7''' \\
k_{88} &= \frac{c^3}{12}D_{22}(\gamma_6''')^2 + 4cD_{66}(\gamma_6')^2 \\
k_{89} &= \frac{c^3}{12}D_{22}\gamma_6'''\gamma_7'' + 4cD_{66}\gamma_6'\gamma_7' \\
k_{8,10} &= -2cD_{26}(\frac{c^3}{12})[\beta_{10}'\gamma_6'' + \beta_{10}''\gamma_6'] - 4cD_{16}\beta_{10}'\gamma_6' \\
k_{99} &= \frac{c^3}{12}D_{22}(\gamma_7''')^2 + 4cD_{66}(\gamma_7')^2 \\
k_{9,10} &= -2cD_{26}(\frac{c^3}{12})[\beta_{10}'\gamma_7'' + \beta_{10}''\gamma_7'] - 4cD_{16}\beta_{10}'\gamma_7' \\
k_{10,10} &= 4cD_{11}(\beta_{10}')^2 + 4D_{12}(\frac{c^3}{12})[\beta_{10}''\beta_{10}'] \\
&\quad + \frac{c^5}{80}D_{22}(\beta_{10}''')^2 + 4D_{66}(\frac{c^3}{12})(\beta_{10}')^2
\end{aligned}$$

An examination of these expressions clearly reveals warping and other edge effects. These terms are multiplied by the factor  $c^3/12$  or  $c^5/80$ . They are not included in conventional elements. Note that the  $D_{22}$  term adds torsional stiffness to the (3,3) and (8,8) terms, but only if a nonlinear torsional deformation function is used. The term  $4cD_{66}$  is an equivalent beam GJ stiffness parameter.

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## 6.0 OVERALL CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

In this research program, the phenomenon of constrained warping of composite aircraft wings, resulting from aeroelastically-induced loads, has been investigated. In the preliminary phase (first year) of this research, the interaction between the constrained warping and the statics, free vibration, divergence or flutter of plate models of composite wings without elastic coupling were studied. In addition, a study of the interaction of the constrained warping and the free vibration for composite wings with elastic coupling was initiated. The principal results revealed by these preliminary studies include the following, (i) the constrained warping phenomenon has a very significant influence on the static, free vibration and aeroelastic stability of aircraft wings fabricated of composite materials, (ii) this influence may be conservative or nonconservative depending on the configuration of the wing, (iii) there exist closed-form solutions to the problem of free aeroelastic oscillations of composite aircraft wings with constrained warping and elastic coupling, (iv) these newly discovered closed-form solutions were instrumental in formulating a physical explanation (which has been non-existent until now) of the vibrational "modal transformation" phenomenon in models of composite aircraft wings.

These results therefore have been able to provide answers to some basic questions about the fundamental nature of the warping phenomenon. However, these new results have raised new questions as well: For example, does the existence of closed form solutions just established for the wing's free oscillations imply the existence of closed-form solutions for the divergence or flutter problems? Other questions include: is it possible to formulate a numerical super-element which can accurately capture the essence of these dynamic

phenomena which can be used for analyzing more complex models/configurations of composite aircraft wings?

Some of the issues have been addressed in a preliminary fashion during the current phase of the research program. In a future research program, therefore, it should be desirable to deal with these issues in greater depth by, (a) completing the analysis of the elastically coupled free vibration with constrained warping and, (b) investigating the elastically coupled divergence and flutter of composite wings with constrained warping, (c) formulating a numerical extended beam superelement, which can capture the essence of constrained warping, for studying geometrically more arbitrary aircraft wing configurations.

## 7.0 STATUS OF PUBLICATIONS

The significant results obtained thus far in this research program are being compiled and written up for publication in suitable technical journals. AIAA Paper No. 86-1006, a paper based on this research has been prepared and was presented at the AIAA/ASME/ASCE/AHS 27th Structures, Structural Dynamics and Materials Conference held in San Antonio, Texas, May 19-21, 1986.

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## 9.0 INTERACTIONS WITH AIR FORCE AGENCIES

Dr. Sinclair M. Scala, Vice President for Research and Dr. Gabriel A. Oyibo, the Principal Investigator and Program Manager for the research program, were invited by Dr. James Olsen, Assistant for Structures and Dynamics Division, FDL, WPAFB on 13 August 1985 to brief him on new research developments at FRC in Aeroelasticity and Transonic aerodynamics. Dr. Olsen and his colleagues were happy with the briefing which included the presentation of some of the results included in this report. They expressed interest in expanding the research program to emphasize its application aspects for the ATF.

## 10.0 DISCOVERIES AND INVENTIONS

The main discoveries of this research program thus far include the following:

- (1) proper modelling of constrained warping phenomenon is very important in aeroelastic analysis, (2) this phenomenon is particularly important for wings with coupling and higher oscillation frequencies, (3) there exist closed-form dynamic solutions for composite wings with elastic coupling and warping constraint; (4) the mechanism for dynamic modal transformations in composite wings may be linked to some conservative inter-modal energy transfer and
- (5) chordwise bending is more important for bending modes than twisting modes for or orthotropic wings.

END

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